A REAL-TIME SIMULATION OF A HUMAN-CONTROLLED, COMMAND-GUIDED, AIR-TO-SURFACE MISSILE SYSTEM

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THESIS

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ABSTRACT

This paper investigates the feasibility of an air-to-surface missile delivery system employing impact prediction to assist a human operator in controlling the system. The impact prediction information was used to drive a display which the operator used in controlling the flight of the weapon. It was desired to find whether impact prediction proved useful in weapons control.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION AND STATEMENT OF THE PROBLEM

A. TYPICAL ASM WEAPONS SYSTEMS

Most older air-to-surface weapons share the characteristic that once the weapon is launched it is under the influence of only the environment, e.g., gravity, wind drift, drag, etc. Examples in this category of weapons are bullets, general-purpose bombs, and rockets. This makes it the responsibility of the weapons officer to insure that the fire-control solution is accurate enough for the weapon to impact at the target within the kill radius of the warhead. In the normal tactical situation this is a difficult task to perform properly. As a result some of the newer weapons systems have been designed with a capability for guidance of the weapon to the target.

These newer systems require that to provide guidance for the weapon, the weapons control officer must maintain visual contact with the weapon during its flight. Again, this solution to the problem suffers from tactical restrictions. In a combat environment it is very difficult to maintain a flight path which allows the weapons control officer to maintain visual contact with the weapon during its flight.

In order to overcome the shortcomings of these systems, even newer systems such as the Phoenix employ very sophisticated missiles which allow the launching aircraft to stand-off from the target and still deliver a weapon so as to destroy or seriously damage the target. One aspect of these systems

which is objectionable is the high cost of the missiles which are employed. As a result this study was made of a system which would employ a stand-off delivery capability while employing a relatively unsophisticated, and therefore inexpensive missile to deliver the warhead.

B. ANALYSIS OF STAND-OFF WEAPONS DELIVERY

Stand-off weapons delivery is best defined as the ability to remain outside the destructive range of the enemy's weapons and deliver ordnance on the enemy installation. The advantages of remaining outside the destructive envelope of the enemy's weapons are obvious. The disadvantages of standing off from the target are numerous. The greater the stand-off distance, the more accurate the guidance and system signals must be as well as the knowledge of the target location. This is necessary to provide a reasonable probability of target kill.

In its most basic form the air-to-surface weapons delivery problem consists of two parts: the detection and location of the target by some means, and delivery of a weapon to the target.

The solution which is proposed in this paper is based on three assumptions about what the author feels an air-to-surface weapons system should incorporate. These three assumptions are 1) that the target should be located by some method which does not endanger the launching aircraft; 2) that the primary purpose of air-to-surface missiles is to deliver ordnance on the target, not to carry a large amount of expensive

electronic equipment to be blown to pieces along with the target; and 3) that to improve the overall accuracy of the system, the missile will be under some form of command guidance during its flight to the target.

The solution which is proposed in this paper is one of the "shoot and loiter" school of weapons control. Those of this line of thought contend that it is more economical to shoot a relatively unintelligent missile at the target and maintain the sophisticated system parts in the launching aircraft. This requires that the launching aircraft remain in the target area until the weapon has hit the target which increases the probability of the launching aircraft being destroyed.

The opposite point of view is that the aircraft should launch the missile and get out of the target area. This approach necessitates the use of a very sophisticated missile but reduces the probability of the launching vehicle being destroyed.

The trade-off to be considered between the two approaches is to obtain the proper balance of economy versus probability of survival of the launching aircraft.

II. THE PROBLEM: ANALYSIS AND SOLUTION

In this chapter the two divisions of the problem will be discussed in detail. The first is the problem of target location. The second, delivery and guidance of the weapon, will be subdivided into two sections. The first section will be concerned with the initial fire-control solution for the missile flight path and the second with the guidance of the missile to the target.

A. LOCATING THE TARGET

The task of locating the target will be limited to the case of a radiating target such as a radar. This limitation was made in order to propose a solution in which the launching aircraft both locates and destroys the target. The assumption of a radiating target allows the use of a direction-finding method to locate the target.

Several methods of direction finding are available. These include triangulation of radiation given off by the target using two or more receivers at known locations, the use of filtered sequential measurements of bearing and depression angles from a single aircraft at known positions, or even the simple method of having two ground observers locate the target. For this solution the second method was used.

1. Basic Geometry of Target Location

In Fig. 1 the basic principle of location by triangulation is illustrated. The points 1 and 2 could be either

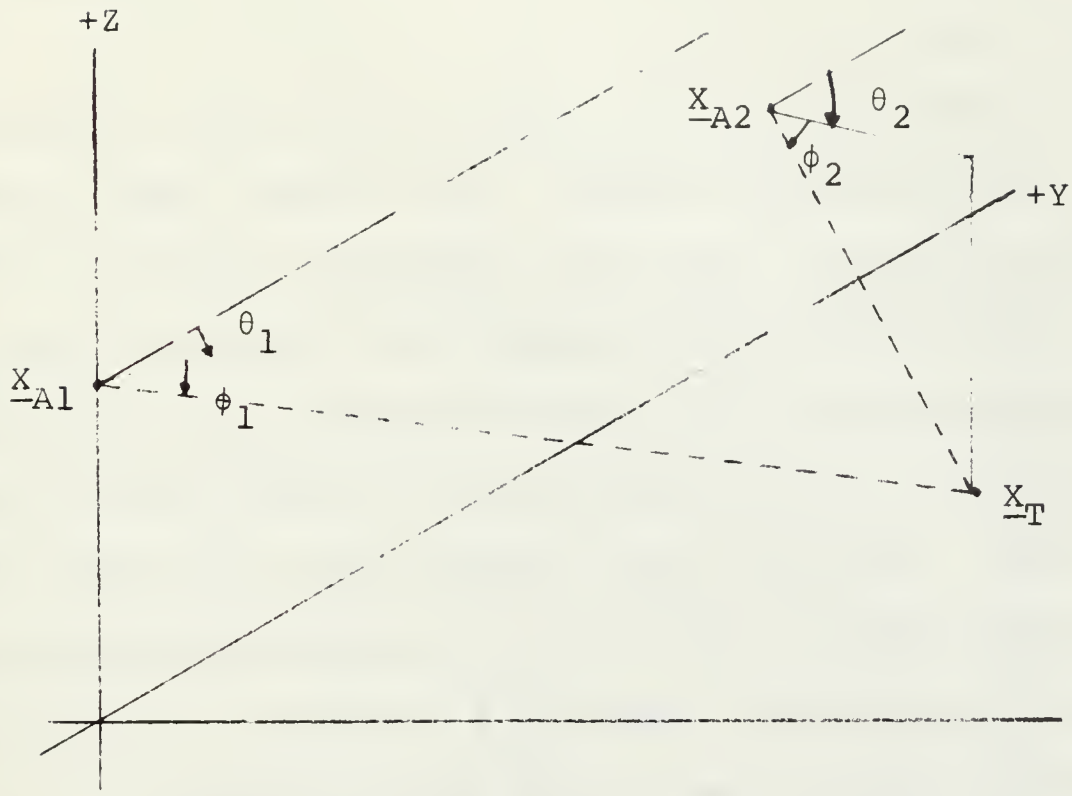

Figure 1. Target Location by Triangulation

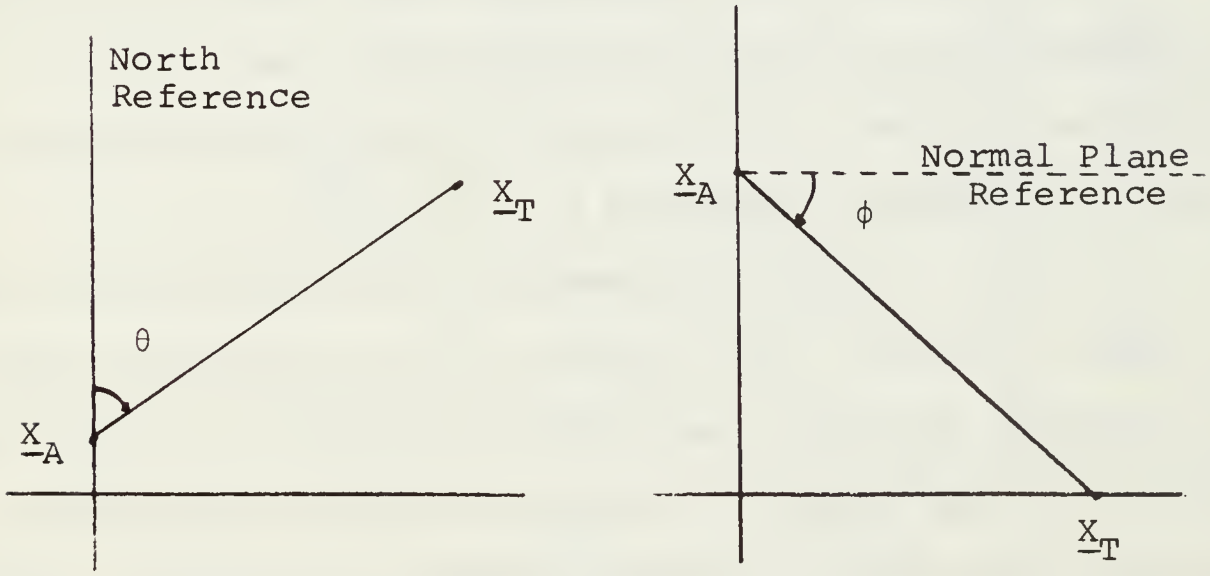

Figure 2. Definitions of Azimuth and Depression Angles

two distinct receivers listening to a signal at the same time, or they could be the two locations of a single aircraft at different times.

Figure 2 gives a definition of the angles involved in all the target-location schemes. The azimuth angle is that angle measured from a true north reference positive clockwise to the line-of-sight (LOS) between the aircraft and the target. The depression angle is the angle measured from a reference plane normal to a line connecting the aircraft and the center of the earth. An angle which rotates below the normal plane is considered negative and one which rotates above the plane is considered positive. Appendix E contains a discussion of the reference systems used in the entire problem.

In the following two sections two different methods of location of the target by use of angular information will be presented.

2. <u>Location Using Positional and Angular Information</u>

The first of the two methods for target location will be explored here for the two-dimensional case to show the principle involved. An analysis of the three-dimensional case is contained in Appendix A.

Figure 3 shows the geometry of the two-position method of target location. The aircraft, at two different times and at different positions, measures the azimuth angle from the aircraft to the target. At the same time as the angle is measured, the position of the aircraft is recorded. Assume

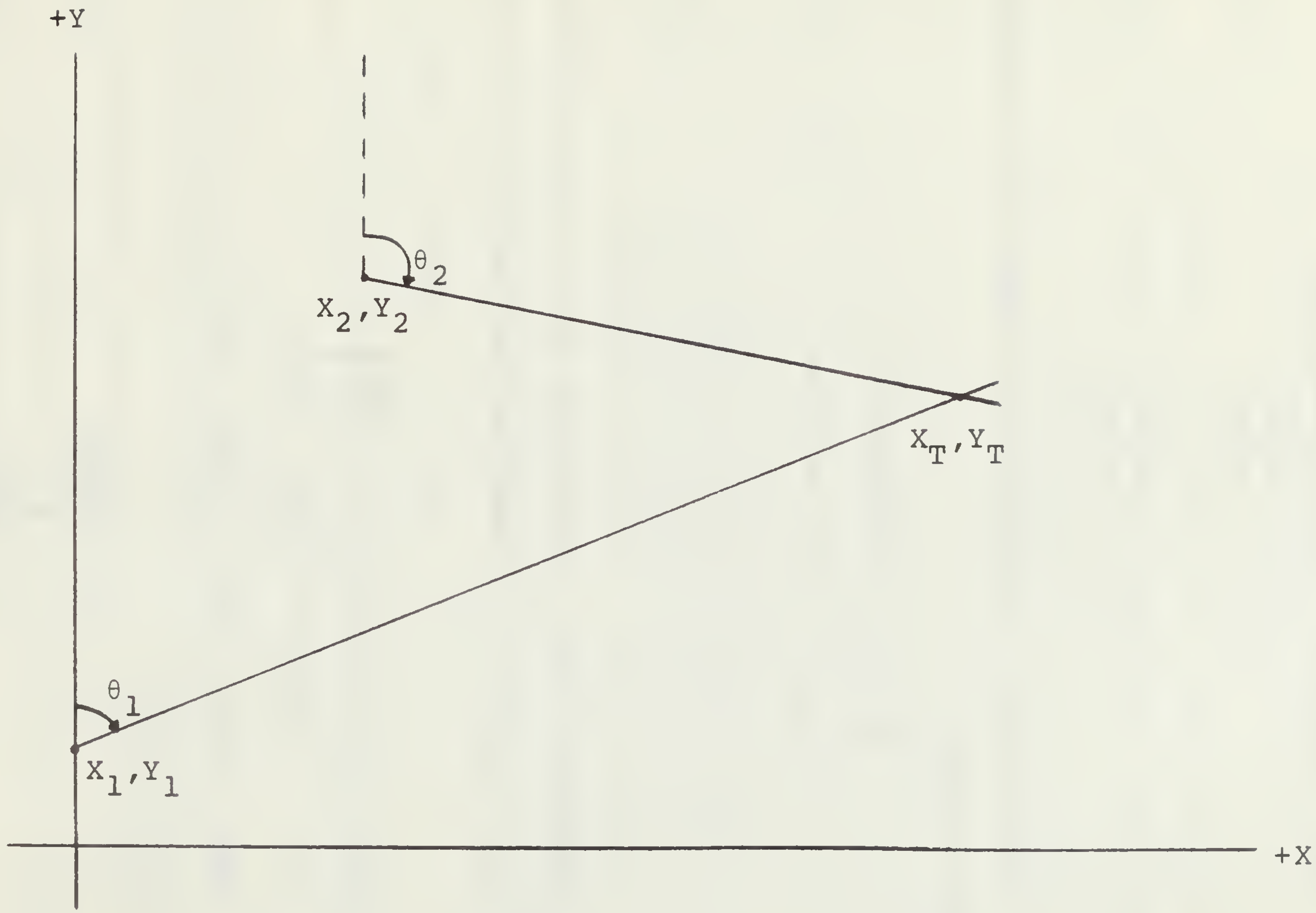

Figure 3. Two-Position, Planar Target Location

that the target coordinates are X_t and Y_t. The measurements are listed below.

Time	Aircraft Location		Azimuth Angle
1	X_1	Y_1	θ_1
2	X_2	Y_2	θ_2

From Figure 3 it can be seen that

$$\text{Tan } \theta_1 = \frac{X_t - X_1}{Y_t - Y_1} \tag{II-1}$$

$$\text{Tan } \theta_2 = \frac{X_t - X_2}{Y_t - Y_2} \quad . \tag{II-2}$$

Writing the two equations as simultaneous equations and solving them:

$$\begin{aligned} X_t - X_1 &= (Y_t - Y_1) \text{ Tan } \theta_1 \\ X_t - X_2 &= (Y_t - Y_2) \text{ Tan } \theta_2 \end{aligned} \tag{II-3}$$

$$\begin{aligned} X_t - Y_t \text{ Tan } \theta_1 &= X_1 - Y_1 \text{ Tan } \theta_1 \\ X_t - Y_t \text{ Tan } \theta_2 &= X_2 - Y_2 \text{ Tan } \theta_2 \quad . \end{aligned} \tag{II-4}$$

Letting $A = \text{Tan } \theta_1$, $B = \text{Tan } \theta_2$, $C = X_1 - Y_1 \text{ Tan } \theta_1$, and $D = X_2 - Y_2 \text{ Tan } \theta_2$ and writing the equations in matrix form

$$\begin{bmatrix} 1 & -A \\ 1 & -B \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix} \quad . \tag{II-5}$$

Using normal matrix algebra techniques, the solution can be shown to be

$$X_t = \frac{DA-BC}{A-B} \tag{II-6}$$

$$Y_t = \frac{D-C}{A-B} \quad . \tag{II-7}$$

Expanding the results into their full form by replacing the A, B, C, and D with their respective values gives

$$X_t = \frac{\mathrm{Tan}\theta_1(X_2-Y_2\mathrm{Tan}\theta_2)-\mathrm{Tan}\theta_2(X_1-Y_1\mathrm{Tan}\theta_1)}{\mathrm{Tan}\theta_1-\mathrm{Tan}\theta_2} \qquad \text{(II-8)}$$

and

$$Y_t = \frac{(X_2-Y_2\mathrm{Tan}\theta_2)-(X_1-Y_1\mathrm{Tan}\theta_1)}{\mathrm{Tan}\theta_1-\mathrm{Tan}\theta_2} . \qquad \text{(II-9)}$$

This technique of locating the target can be quite effective and accurate if the two positions used in deriving the solution are far enough apart and if the bearing angle measurements are respectably accurate. Ideally the two bearing angles should be separated by ninety degrees. This separation will reduce to a minimum the effects of any noise in the measurement of the angles. If the target radiates for only short periods of time and the ambient conditions are such that the measurements are very noisy, the derived target location could be greatly in error. For this reason an alternate approach to the problem of locating the target was taken. The alternate method uses only angular information and a hybrid filtering technique.

3. Location Using Angular Information and Hybrid Filtering

The target location method discussed in this section is the result of research done by Demetry [1]. The method employs an unusual hybrid filtering technique in which a Kalman filter is considered linear for calculation of a gain schedule but in which prediction is done nonlinearly. The hybrid filter will be discussed here briefly.

It is desired to locate a radiating target by a method in which only a bearing angle to the target is measurable. The equations below give the relationships used in the formulation of the filter.

$$\underline{\hat{a}}_{k|k} = \underline{h}(\hat{\theta}_{k|k}, \hat{\dot{\theta}}_{k|k}, \underline{\xi}_k) \qquad \text{(II-10)}$$

where

$$\underline{\hat{a}} = \begin{bmatrix} \widehat{XT}_{k|k} \\ \widehat{YT}_{k|k} \end{bmatrix} \qquad \text{(II-11)}$$

is the estimated target location at time k and

$$\underline{\xi}_k = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ y(t) \\ \dot{y}(t) \end{bmatrix} \qquad \text{(II-12)}$$

is the aircraft vector.

The aircraft position and velocity components are considered to be measurable without noise for this evaluation as they are throughout the remainder of the problem.

The bearing angle dynamics are described by the differential equations

$$\underline{\psi}(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \underline{f}(\underline{a}, \underline{\xi}(t), t) \qquad \text{(II-13)}$$

where $\underline{f}$ is a nonlinear vector function. The angle measurement at any discrete time k is

$$Z_k = H\underline{\psi}_k + v_k, \quad k = \frac{t}{T} \qquad \text{(II-14)}$$

in which H = [1 0 0] and v_k is the error in the bearing angle. The v_k is random in nature with an assumed mean value of zero.

The gain schedule was calculated by assuming linear dynamics and constant acceleration for the process and using

the recursive Kalman gain equations [2]. However, the prediction of angular information was done by using the actual angular dynamics in a discretized form rather than employing the linear Φ matrix which was used in computing the filter gains. These equations, by using true system dynamics to make approximate predictions, reduce the location error since, in effect, they consider the system linear. In this system this is not a bad assumption since the sample period is relatively short compared to the rates of change of the system variables.

$$\hat{\theta}_{k+1|k} = \tan^{-1}\left[\frac{\widehat{XT}_{k|k}-x_k}{\widehat{YT}_{k|k}-y_k-\dot{y}_k T}\right] \tag{II-15}$$

$$\hat{\dot{\theta}}_{k+1|k} = \frac{\dot{y}_k[\widehat{XT}_{k|k}-x_k]}{[\widehat{YT}_{k|k}-y_k-\dot{y}_k T]^2+[\widehat{XT}_{k|k}-x_k]^2} \tag{II-16}$$

$$\hat{\ddot{\theta}}_{k+1|k} = \frac{2\dot{y}_k{}^2[\widehat{XT}_{k|k}-x_k][\widehat{YT}_{k|k}-y_k-\dot{y}_k T]}{\{[\widehat{YT}_{k|k}-y_k-\dot{y}_k T]^2+[\widehat{XT}_{k|k}-x_k]^2\}^2} \quad . \tag{II-17}$$

This is a brief discussion of this particular method of location of a target. For the reader interested in pursuing this further, Ref. 1 will be of interest.

B. THE FIRE-CONTROL SOLUTION

Before it is possible to launch a missile at a target it is necessary to develop a solution for a flight path which will take the missile from the launch point to the target. The accuracy with which the solution need be computed is entirely a function of the amount of guidance which will be available to influence the missile during its flight. Also,

before an initial solution can be computed, it is necessary to know what approximate flight path the missile will follow from the launch point to the target.

In the solution which is being investigated here, the proposed flight path for the missile is a nominal parabolic path. The path the missile actually travels will not be parabolic due to air drag, control effects, and other outside influences.

There are two main reasons for selecting a parabola as a nominal flight path. The first concerns a relationship between the height of a parabola and distance to the target from a point under the peak of the parabola. This ratio is referred to as the A/R ratio and its importance is discussed in Appendix B. The second consideration is that the missile will be under the influence of gravity in a very helpful manner. The gravitational force will help to maintain the velocity of the missile and hence, the controllability of the missile, after the boosted phase of the missile's flight.

1. <u>Geometry of the Fire Control Solution</u>

The general geometry of the fire control problem is illustrated in Fig. 4. In the solution the aircraft, point A/C on the figure, is considered to be at the X origin of the figure and the target, point T in the figure, is considered to lie on the X-axis. The parabola will be defined such that it contains the two points of the launching aircraft (A/C) and the target (T). Since a parabola is defined by a second-order polynomial, e.g., of the form $y = ax^2 + bx + c$, some third point on the parabola must be defined.

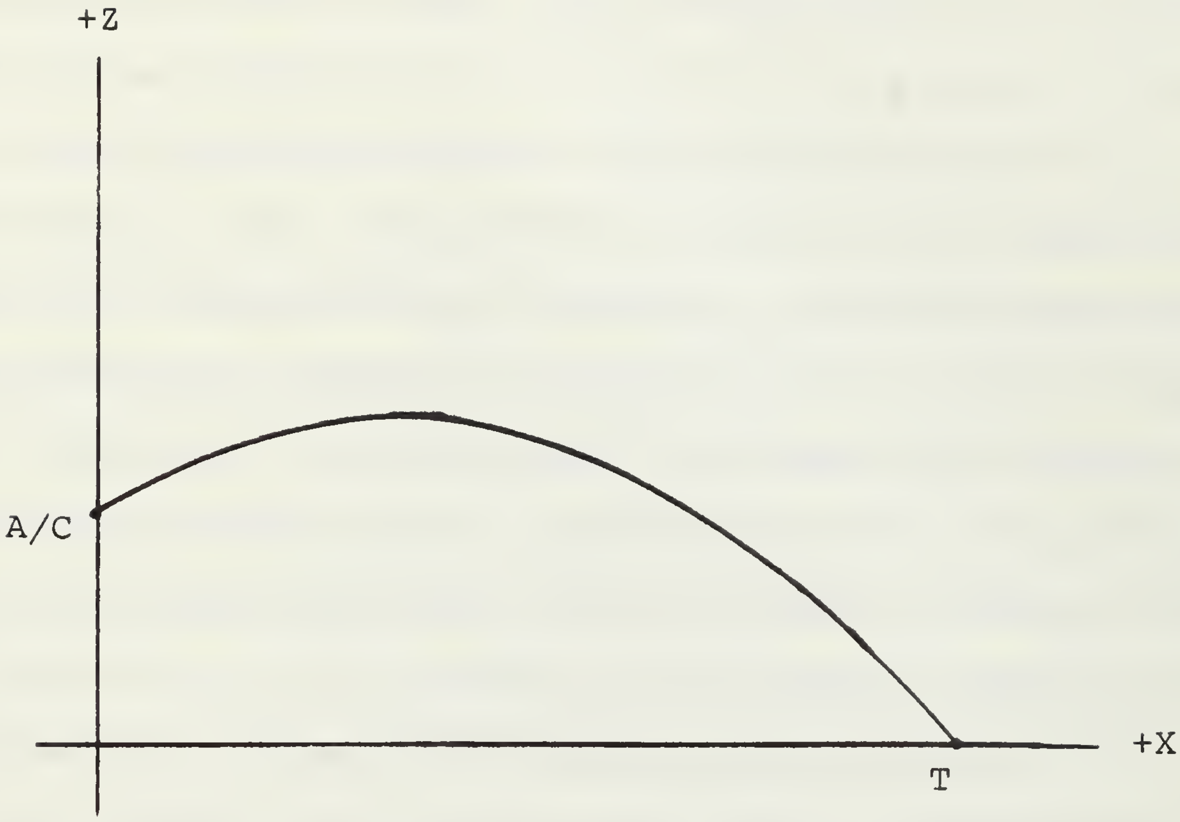

Figure 4. The Nominal Parabolic Flight Path

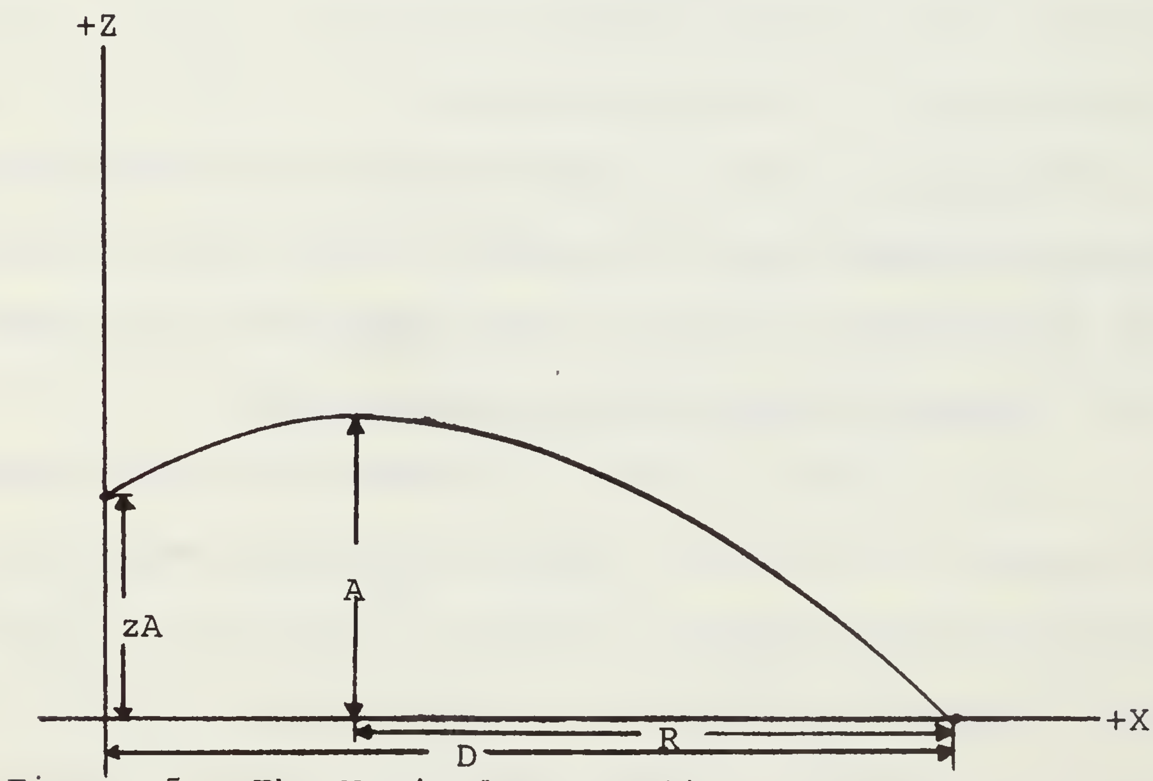

Figure 5. The Nominal Parabolic Flight Path with Distances Defined

In Fig. 5, the parabola of Fig. 4 is redrawn, this time with three points and four distances defined. For the purpose of defining the nominal parabola, the launching aircraft is placed on the Z axis so that range is zero at the origin. The altitude of the aircraft is labeled zA. A second point is the location of the target. The target is at zero elevation and a distance D from the origin. The third point, which is necessary for the parabola to be defined, is the peak of the parabola. The altitude of this point is A and it is D - R from the origin. Using these three points it is possible to solve for the coefficients of the second-order polynomial defining a parabola.

Writing the equations of the parabola at the three points gives

$$a(0)^2 + b(0) + c = zA \qquad \text{(II-18)}$$

$$a(D-R)^2 + b(D-R) + c = A \qquad \text{(II-19)}$$

$$a(D)^2 + b(D) + c = 0 \qquad \text{(II-20)}$$

$$R = kD \quad . \qquad \text{(II-21)}$$

Equation II-21 is used to define the relationship between the range from the origin to the target and the range from under the peak of the parabola to the target.

2. Method of Obtaining the Launch Angle

In a system in which the launch point and target point are coplanar in the earth plane, a ballistic mass will travel furthest when the velocity vector is initially directed 45 degrees above the horizontal. This is true as long as the mass has no external forces acting on it other than gravity.

Two factors involved in this investigation make the choice of a 45-degree launch angle unacceptable. The first is that the launching point and target are not earth coplanar and, secondly, that the mass is subject to thrust from a rocket motor. Thus, the problem of computing the launch angle becomes one of maximizing the range as a function of the distance between the target and launching point in the Z plane.

The results of this particular maximization were interesting. For aircraft locations which were up to 5000 meters noncoplanar with the target, the launch angle for maximum distance was found to vary only one degree, between 47 and 48 degrees. (The launch angle as used here is defined as the angle between the velocity vector and the horizontal at the launch point. This is not the angle of the velocity vector at the time of burnout which is the angle which determines the maximum distance of flight.) The launch angle was set at a constant 47.5 degrees as a result. This angle should give the maximum range regardless of range to the target. Starting with the launch angle that gives maximum range should prove entirely acceptable since one of the assumptions about the system was that the missile would be under some type of guidance during its flight.

C. GUIDANCE BY THE HUMAN OPERATOR

Throughout this paper mention has been made of the fact that the missile will be guided by the human operator throughout its flight. In actuality the missile was not released to the operator's control until after it had finished the boosted

phase of its flight. During this time the system turned the missile up to the launch angle and yawed it to the proper flight heading. Once the missile started its free flight, a display of the predicted impact point of the missile relative to the target was generated for the missile control officer (MCO). At the same time, the control surfaces of the missile were set to a neutral position and control of the missile was given to the MCO who then guided the missile the remainder of the distance to the target.

In order for the predicted missile impact point to be displayed, it was first necessary to track the missile and based on the tracking information, predict its impact point. The prediction information was then presented to the MCO in such a manner that he could make decisions as to the guidance control to be given to the missile.

1. Tracking the Missile in Flight

In this problem the missile was tracked in flight by a radar which was capable of providing four items of information. These were the range to the missile, the range rate of change to the missile, the azimuth angle to the missile, and the depression angle to the missile. The reference system by which the angles were measured is discussed in Appendix E. The range was measured in the positive sense while the range rate was considered positive for a missile whose range was increasing and negative for a missile which was closing the launching aircraft.

Since the radar was operating in a natural environment, the information provided by the radar was not perfect. This noise would introduce errors in the estimation of the location of the missile during its flight, so it was desirable to use some technique to reduce the effect of the noise on the measurements. The technique used was Kalman filtering. Before deriving the necessary Kalman filter for this problem, the desired missile information vector should be investigated.

Figure 6 presents the geometry for developing the missile information vector from the spherical vector of the radar. The desired missile information vector contains six components: the XYZ cartesian location of the missile relative to the aircraft and the XYZ cartesian velocities, again relative to the aircraft. The acceptability of providing only a six-element state vector for the missile is dependent upon the sample rate at which the state vector is developed. Even though the missile is being acted upon by external forces, it is possible to consider the missile to be a piece-wise constant-velocity missile in either spherical or cartesian coordinates if the interval between samples is sufficiently small.

From Fig. 6 the equations for the missile location relative to the aircraft can be developed. Then by the use of the time derivatives of the equations of location, the equations of the cartesian velocities can be found. First, the equations of location.

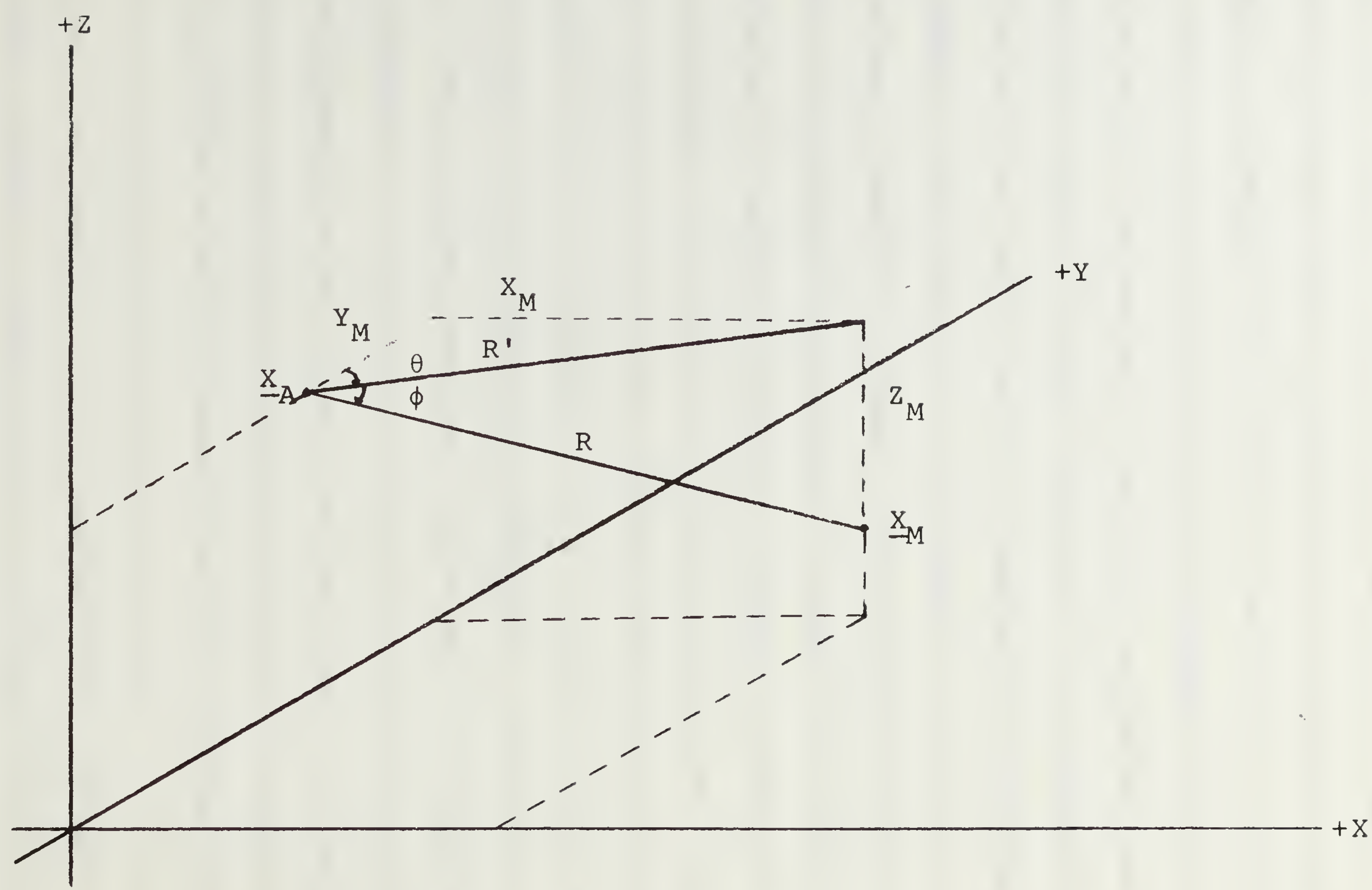

Figure 6. Geometry of Missile Location

$$Z_m = R \sin \phi \tag{II-22}$$

$$X_m = R' \sin \theta = R \cos \phi \sin \theta \tag{II-23}$$

$$Y_m = R' \cos \theta = R \cos \phi \cos \theta \tag{II-24}$$

For the case shown in Fig. 6, the angle ϕ is negative, giving the proper negative value for Z_m. Likewise, the signs of the remaining two values will be correct depending on the quadrant of the angle θ.

To form the equations of the velocities in the cartesian frame, it is necessary only to take the time derivatives of equations II-22, II-23, and II-24. These are given below.

$$\dot{Z}_m = R \cos \phi \, \dot{\phi} - \dot{R} \sin \phi \tag{II-25}$$

$$\dot{X}_m = R \cos \phi \cos \theta \, \dot{\theta} - R \sin \phi \sin \theta \, \dot{\phi} + \dot{R} \cos \phi \sin \theta \tag{II-26}$$

$$\dot{Y}_m = -R \cos \phi \sin \theta \, \dot{\theta} - R \sin \phi \cos \theta \, \dot{\phi} + \dot{R} \cos \phi \cos \theta \tag{II-27}$$

From equations II-22 through II-27 a list of the necessary spherical values of missile movement to develop the cartesian state vector can be collected. These equations show that it is necessary to have a vector of spherical values of range, azimuth, depression, and the associated rates of change of the three quantities. From this requirement a Kalman filter can be developed which will not only reduce the effect of the measurement noise on the quantities which are being measured, but also provide estimates of the angular rates which are not measured.

The Kalman filter for filtering and estimating the radar vector has the general Kalman predictor-corrector form

of equation II-28. The G_k is computed from the recursive equations of II-29, II-30, and II-31. More complete details of the derivation of the filter and the values used in the gain schedule computation are given in Appendix C.

$$\hat{\underline{X}}_{k|k} = \hat{\underline{X}}_{k|k-1} + G_k(z_k - H\hat{\underline{X}}_{k|k-1}) \tag{II-28}$$

$$G_k = P_{k|k-1}H^t(HP_{k|k-1}H^t + R)^{-1} \tag{II-29}$$

$$P_{k|k} = (I - G_kH)P_{k|k-1} \tag{II-30}$$

$$P_{k+1|k} = \Phi P_{k|k}\Phi^t + Q \tag{II-31}$$

2. Predicting the Missile Impact Point

In predicting the missile impact point it is desirable to have an algorithm which is fairly accurate. This is the case since it is the point at which the missile is predicted to strike that will be used by the MCO to develop his guidance commands. A requisite increase in accuracy over a ballistic model can be obtained by introducing the effects of drag into the equations of motion for the missile. For the prediction algorithm developed here, the drag was considered to be proportional to the velocity of the missile.

In this solution the drag coefficient of the missile is taken to be constant from the time of the prediction until the missile strikes the target. This, of course, is an invalid assumption but one which is necessary in order to develop a closed-form solution for the prediction equations. The error which is introduced by this assumption will have the effect of causing the MCO to hold the missile aloft longer than would normally be necessary. This action by the MCO will tend to increase the A/R ratio, thus making the missile impact on the

target more vertically and hence less subject to error from target altitude location error. (See Appendix B).

a. Equations of Motion for Impact Prediction

The equations of motion for impact prediction were written for each of the three component directions of the cartesian reference system. The equations were based on an assumed point-mass model of the missile subjected only to drag and gravity forces. The X and Y equations are similar to each other, varying only in the drag coefficient. The Z equation takes into account the effect of gravity. The equations of motion are

$$m\ddot{X} = -K_1\dot{X} \tag{II-32}$$

$$m\ddot{Y} = -K_2\dot{Y} \tag{II-33}$$

$$m\ddot{Z} = -K_3\dot{Z} - mG \tag{II-34}$$

where

$$K_1 = \rho|\dot{X}|A_r \tag{II-35}$$

$$K_2 = \rho|\dot{Y}|A_r \tag{II-36}$$

$$K_3 = \rho|\dot{Z}|A_r \tag{II-37}$$

and m = missile mass, G = gravity, and A_r = reference area.

The method for predicting the impact point of the missile is to first compute the drag coefficients (K_1, K_2, and K_3) which are then considered constant until impact. The Z motion equation is then solved for the time when the missile Z location is equal to the target Z. First, it is necessary to solve equation II-34 for the missile Z as a function of time.

b. Impact Prediction Equations

The result of solving for the Z motion as a function of time is

$$Z(t) = (1-A)e^{-\alpha t} - Bt + C \qquad \text{(II-38)}$$

where

$$A = \frac{m\dot{Z}(0)}{K_3} + \frac{m^2 G}{K_3^{\,2}} \qquad \text{(II-39)}$$

$$B = \frac{mG}{K_3} \qquad \text{(II-40)}$$

$$C = Z_m(0) \qquad \text{(II-41)}$$

$$\alpha = \frac{K_3}{m} \quad . \qquad \text{(II-42)}$$

This mixed equation is not directly solvable for that value of time such that Z_m is equal to Z_t. The Newton-Raphson iterative method is employed to find the value of time which will satisfy the equation for a specific set of conditions (a particular $Z_m(0)$ and $\dot{Z}_m(0)$). The value of time thus found is substituted into the X and Y solutions to find the predicted impact point of the missile. The X and Y predicted positions at impact are given by equations II-43 and II-44.

$$X_p = X_m(0) + \frac{m\dot{X}_m(0)}{K_1}\left(1-e^{-K_1 t_i/m}\right) \qquad \text{(II-43)}$$

$$Y_p = Y_m(0) + \frac{m\dot{Y}_m(0)}{K_2}\left(1-e^{-K_2 t_i/m}\right) \qquad \text{(II-44)}$$

where t_i is the value of impact time from the solution of equation II-38.

3. Display of the Situation

In order for the MCO to make intelligent decisions regarding the application of control to the missile during flight, there had to be some method for displaying the predicted results of the current flight path. What was displayed to the MCO was the current prediction of the impact point of the missile assuming that there would be no further control exercised over the missile. The display is generated such that the center of the screen corresponds to the last estimate of the target position and the predicted impact point is displayed relative to the center of the screen. The display is also rotated so that motion of the impact point will correspond to control stick motion. Figure 7 illustrates the display screen and Appendix H further discusses the man/system interface.

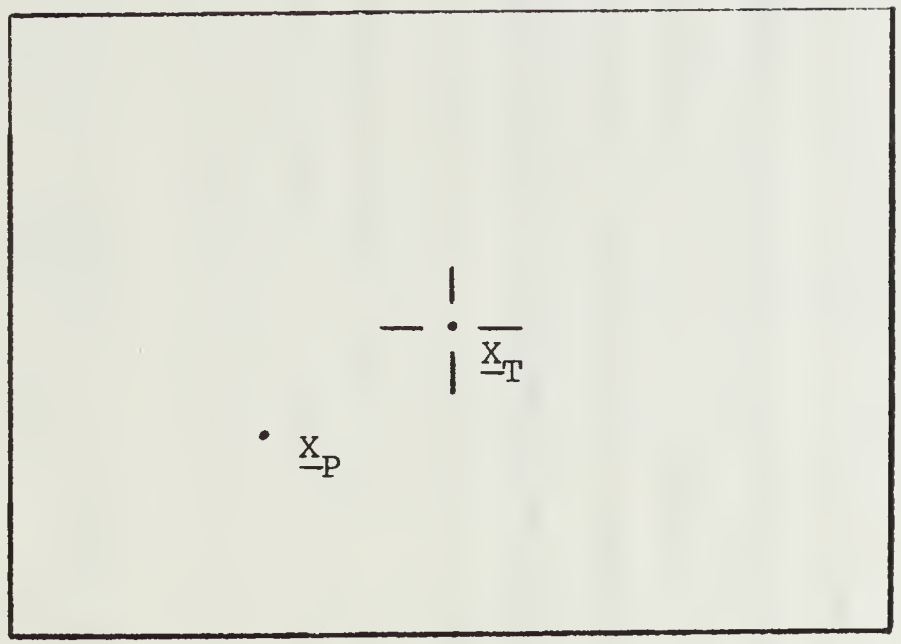

Figure 7. The Situation Display

III. THE SIMULATION OF THE PROBLEM

A proposed solution to a problem cannot be considered to be valid unless it is tested in some way in practice. One method of evaluation is to build an actual system and test it in a practical operating environment. This is generally too expensive to be feasible. Another method of testing a proposal is to simulate the system on an appropriate computer. The second method was chosen to evaluate the problem solution proposed in this paper.

This simulation was done on two different computers. The first was an IBM System 360 Model 67, operating under the MVT operating system. This system was used for initial program debugging since the Operating System has an excellent error-monitoring system. The second system was a hybrid computer consisting of an SDS 9300 digital computer and a COMCOR CI-5000 analog computer.

A. SIMULATION OF TARGET LOCATION

The simulation of the target location was accomplished by using the results of work in passive target location done by Demetry [1]. The results of the hybrid location method discussed in Chapter II were used by making use of the calculated mean-square error values in target X and Y location as a function of time, from the hybrid location algorithm.

Using these values involved a process of both normalization and interpolation. Since the simulation is designed for

a time increment of one second, the target location mean-square errors had to be interpolated since they were computed on a time increment of two seconds between observations.

The interpolation was made on the assumption that the mean-square errors of location were piece-wise linear over the measurement interval involved. The normalization process was necessary since when the values were generated, the aircraft was flying a set course which had a direct effect on the value of the mean-square error. Thus, the values were normalized by multiplying them by a factor which was the closest point of approach (CPA) to the target for the problem divided by the CPA to the target in the mean-square-error computation program.

B. SIMULATION OF THE FIRE-CONTROL SOLUTION

A real-time simulation of the fire-control solution was eliminated after some research into the problem. A constant launch angle was used in its place. The solution was arrived at by assuming the missile to be a drag-free ballistic mass with the propulsion system of the true missile attached. This approximation was used in an iterative method of computing the angle which would give the maximum flight down range for a particular launch altitude. The iteration was performed on both the launch altitude and the launch angle. For this investigation the angular increment was taken as one degree and the altitude increment was fifty meters.

For each launch altitude, the angle which gave the maximum down-range flight of the assumed missile was plotted against

the launch altitude. The results were surprising in that it was found that for altitudes between fifty and five thousand meters, the angle varied only between 47 and 48 degrees. The launch angle is defined as the initial angle between the horizontal and the velocity vector. This initial angle is reduced by the effect of gravity to an angle at burnout which will give the maximum down-range flight of the mass. The force of gravity is thus responsible for the initial angle being greater than 45 degrees.

It must be noted that this angle was chosen to give the maximum down-range flight of the assumed missile. Thus, by shooting the true missile at this angle the maximum possible down-range flight would be possible for zero control. By the application of control to the missile in the pitch mode, this range can be either increased or decreased. In the desired situation this range should have to be decreased by using down pitch. This action would cause an increase in the A/R ratio which has already been demonstrated to be desirable.

C. SIMULATION OF THE MISSILE

The missile model was the most important part of the simulation and thus was treated with the most rigor. The attempt was made to use a fairly accurate model in that dynamics approximating an actual missile were assumed. Appendix F contains a description of the missile model.

D. GUIDANCE SYNTHESIS

The guidance synthesis for the simulation was done in two parts: one part for the simulation while it was executed on the System 360 and the second for the actual data-gathering simulation on the SDS/COMCOR hybrid.

1. System 360 Guidance Synthesis

The guidance synthesis used during the preliminary runs on the System 360 was a primitive, bang-bang control. Yaw was either plus or minus two degrees per second and pitch was a plus or minus 0.5 radian deflection of the control surface. The assumptions on which this control was synthesized were the same as those discussed in the following sections.

2. SDS/COMCOR Guidance Synthesis

Before going into the discussion of the methods used to develop the guidance commands in the hybrid simulation, the assumptions on which the command synthesis was based will be discussed.

a. Roll-Independent Control

The assumption that control is roll independent is valid since the majority of missile control systems operate in this manner. Also the assumption allows a simplification of the command vector synthesis. The missile is thus considered to either be roll stabilized or the control system in the missile is so designed to compensate for any roll of the missile.

b. Two-Component Control Vector

With roll independence assumed, the control vector consists of two components, a pitch and a yaw component. These components were limited in their magnitude by the synthesis program.

The yaw was limited to a maximum rate of twenty degrees per second by the control mechanism itself. The pitch was limited to plus or minus forty-five degrees of control deflection. Additional limits on the control deflections were built into the missile model. Yaw rate was limited so that radial acceleration on the missile would not exceed ten G's.

c. Control Vector Formation

The control vector was formed by the human operator directing a control stick. The maximum deflection of the stick corresponds to the maximum amount of control which could be generated. This control was then applied to the missile each second until the control stick was neutralized. The control affected the missile in that the control surfaces of the missile remained deflected only as long as the stick was held in a non-neutral position.

IV. DISCUSSION

The simulation of this problem was performed only a few times with the completed program. As a result it was not possible to draw concrete conclusions about the validity of the total concept tested. However, several observations were made about the program and some areas where further investigation could be made were discovered.

A. OBSERVATIONS

One observation from the running of the program supported the supposition that the use of impact prediction in an air-to-surface environment was effective. The errors in impact prediction seemed to be related closely to errors in producing the measured missile state vector. Overall the impact prediction approach to missile control appeared very feasible.

Another observation which was made concerned the initial launch angle assumed for the simulation. Apparently the use of a fixed angle of launch intended to give maximum down-range flight was not as good an assumption as first thought. For targets which were close to the launching aircraft, it was possible to overfly the target by shooting for maximum range. It would be much more desirable to have the launch angle be a function of the initial range to the target.

The overfly or fly-by effect can be a serious problem. If the missile bypasses the target with a small angular offset to either side, it is possible that immediate application of yaw control in such circumstances will cause the missile to

circle the target at a constant range. This can occur since the missile is limited in the amount of radial acceleration it can withstand. It should be possible to overcome the problem by the use of tactical doctrine if the missile does overfly the target. One suggested doctrine would be to continue the fly-by course until the missile is far enough out to make the turn and hit the target area. This maneuver is illustrated in Fig. 8.

B. FURTHER INVESTIGATION

Three areas exist which the author feels merit further investigation. The areas are 1) prediction accuracy and methods; 2) radar filter accuracy; and 3) display methods.

The accuracy of impact prediction should be investigated further. Although it was shown that the impact prediction method worked, no study of the accuracy of the method employed was done. Also, it is possible that different methods of impact prediction would prove to be more accurate than the velocity-proportional model used in the simulation.

Prediction accuracy, from all indications, was very dependent on the accuracy with which the missile state was estimated. The key portion of the program in the missile-state estimation was the radar vector filter. The filter used in the problem was based on the assumption of a constant-velocity model of the missile. The use of a filter based on constant acceleration should be investigated as lags were noted in the measured missile state compared to the true missile vector when the missile was making hard or continued maneuvers.

The method of displaying the prediction information to the MCO also deserves further investigation. The display as it was implemented did not provide any indication of the velocity of the missile. It is felt that if the MCO were given missile velocity information, he would be able to better control the flight of the missile. This is especially true in the fly-by case.

As the missile bypasses the target, the display gives a false indication as to the control necessary to hit the target. This problem is generated as a result of the almost 180-degree change in angle by which the display is rotated.

Figure 9 illustrates an improved display which should solve the rotation problems caused by fly-by and also give velocity vector information to the MCO. The display is of the entire area of the problem. The axes of the new display are neither translated nor rotated as they were in the simulation.

Three points are included on the display. One is the current XY measured position of the missile. This is the left-most point in Fig. 9. The other two points are the current predicted impact point and the last estimate of the target location. A line from the missile position to the impact point will indicate the velocity information desired. Also the non-rotation of the display axes should alleviate the false indication of necessary command.

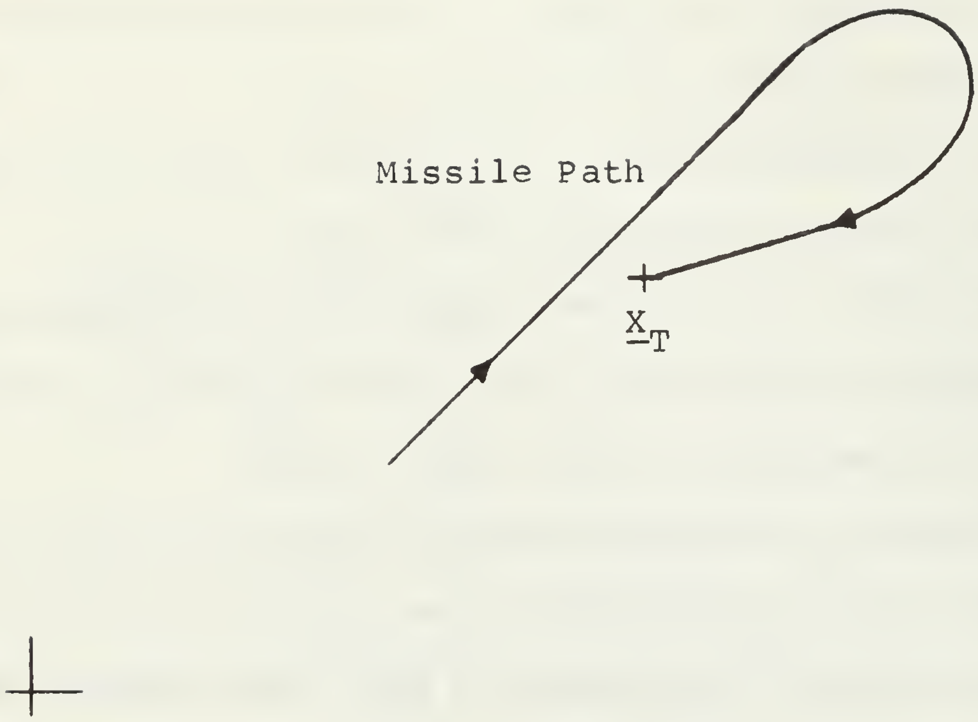

Figure 8. Course for Fly-by Error Correction

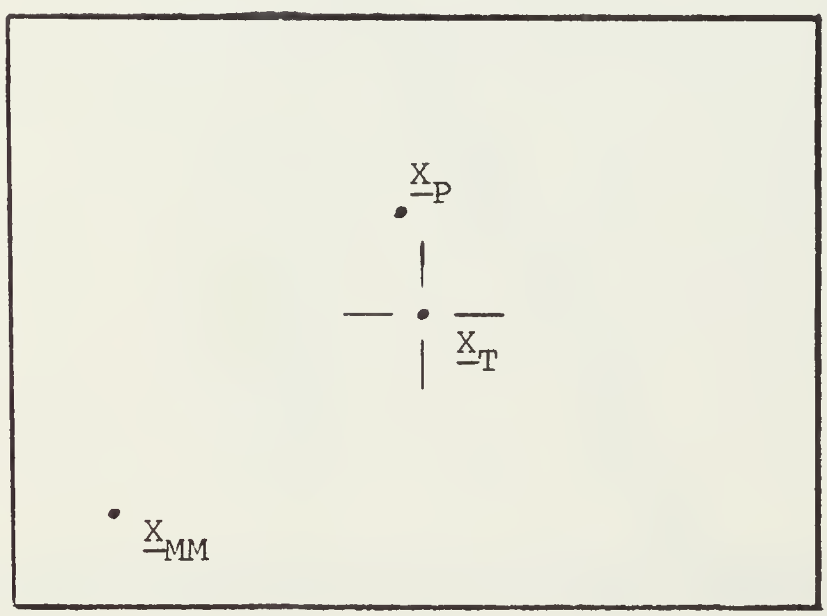

Figure 9. Suggested Improved Display

C. RESULTS OF A RUN

The figures in this section show the results of only one run of the simulation program. The figures give two planar plots of the flight path. One is a plot of the XY plane flight of the missile (Fig. 10) and the second is a vertical plane view of the flight path (Fig. 11).

Other plots are included which show the control vector as a function of time (Fig. 12) and the missile location errors as a function of time (Fig. 13-14).

It should be emphasized again that this is only one run and should not be considered to be in any way conclusive. To reach any definite conclusions a much more exhaustive investigation would have to be made.

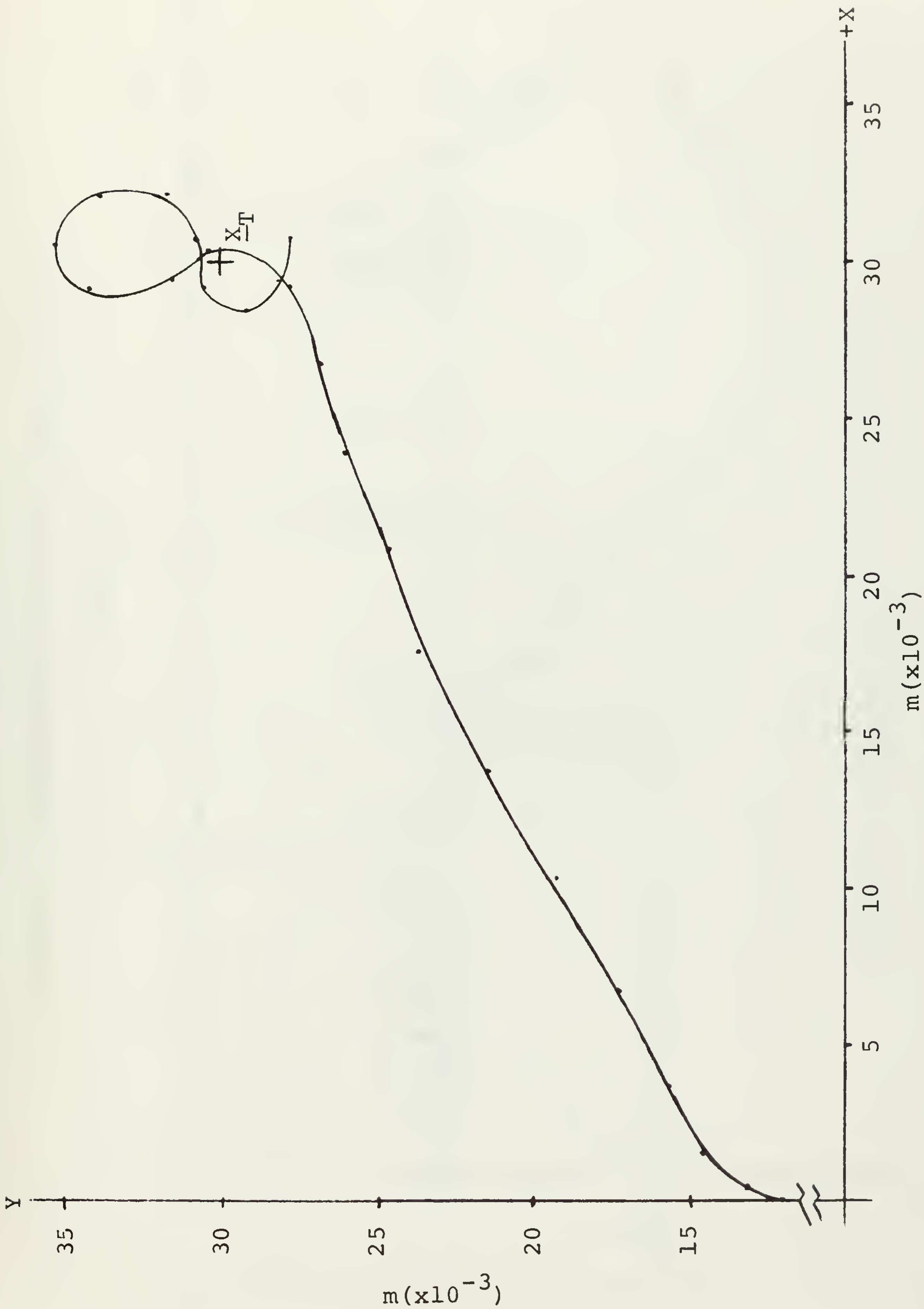

Figure 10. Representative Run XY Flight Profile

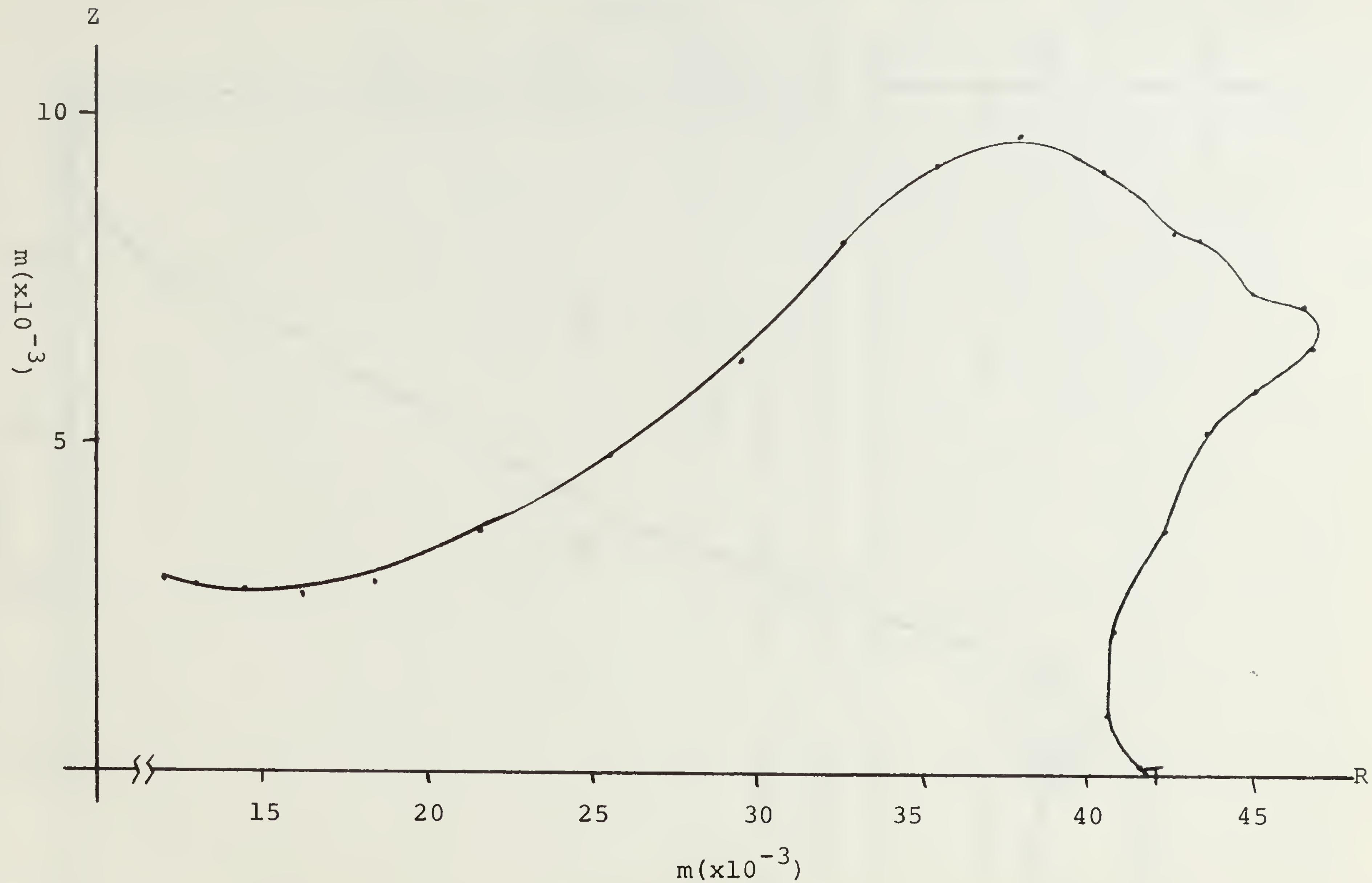

Figure 11. Representative Run RZ Flight Profile

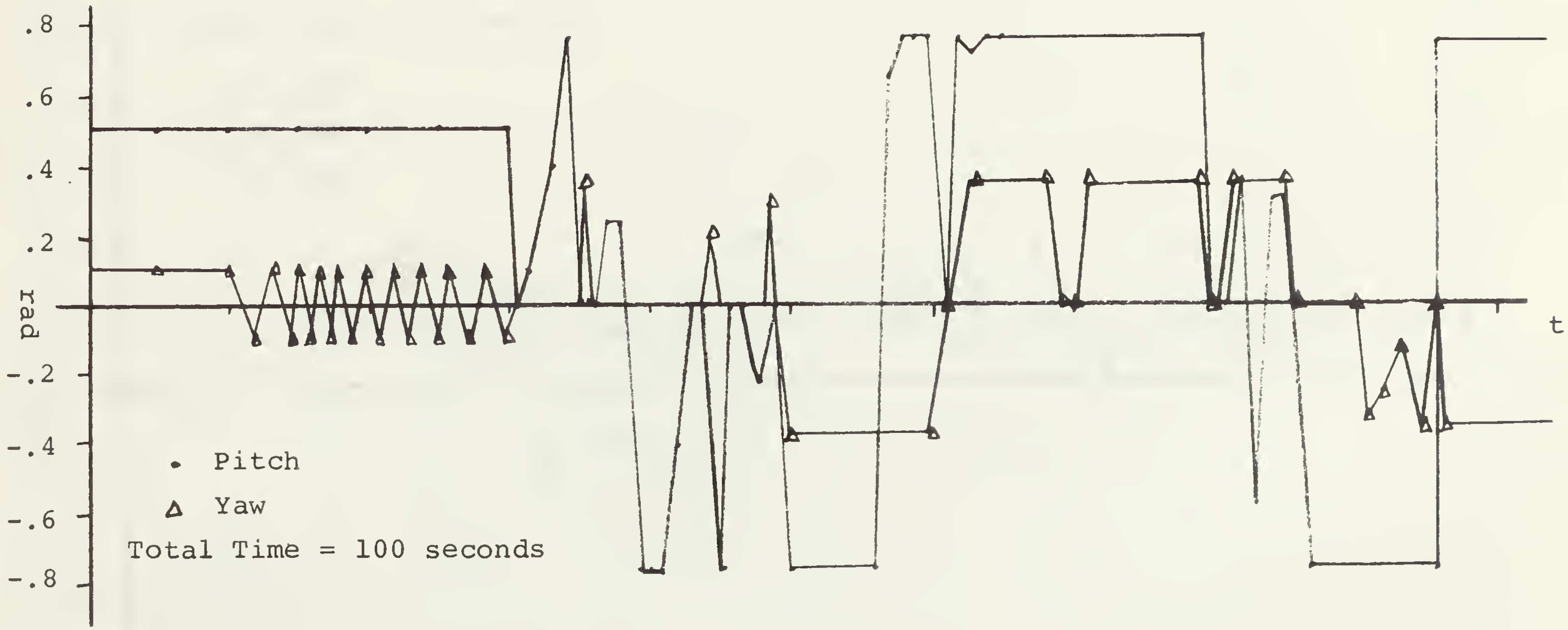

Figure 12. Missile Control vs. Flight Time

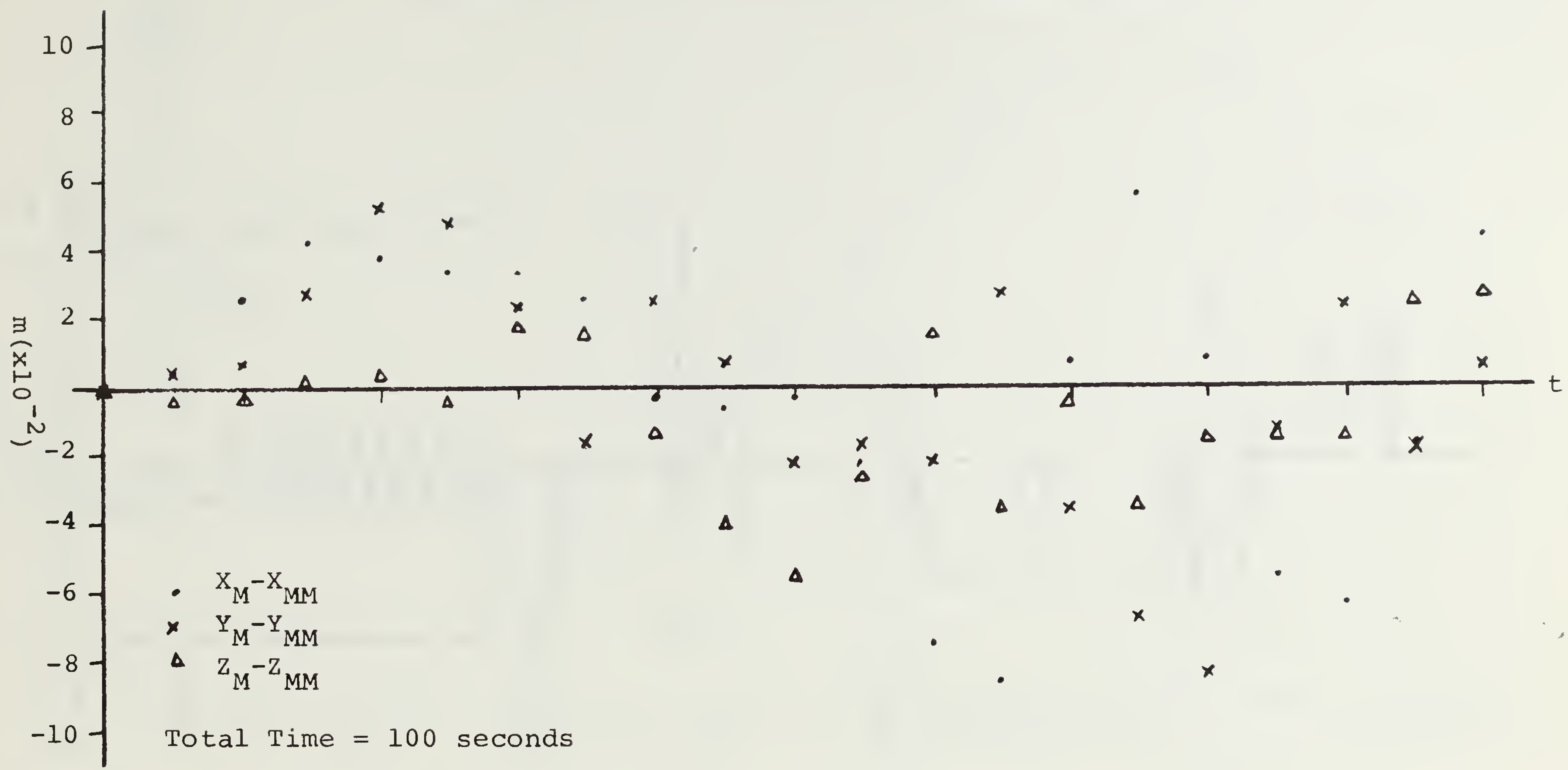

Figure 13. Missile Position Error vs. Flight Time

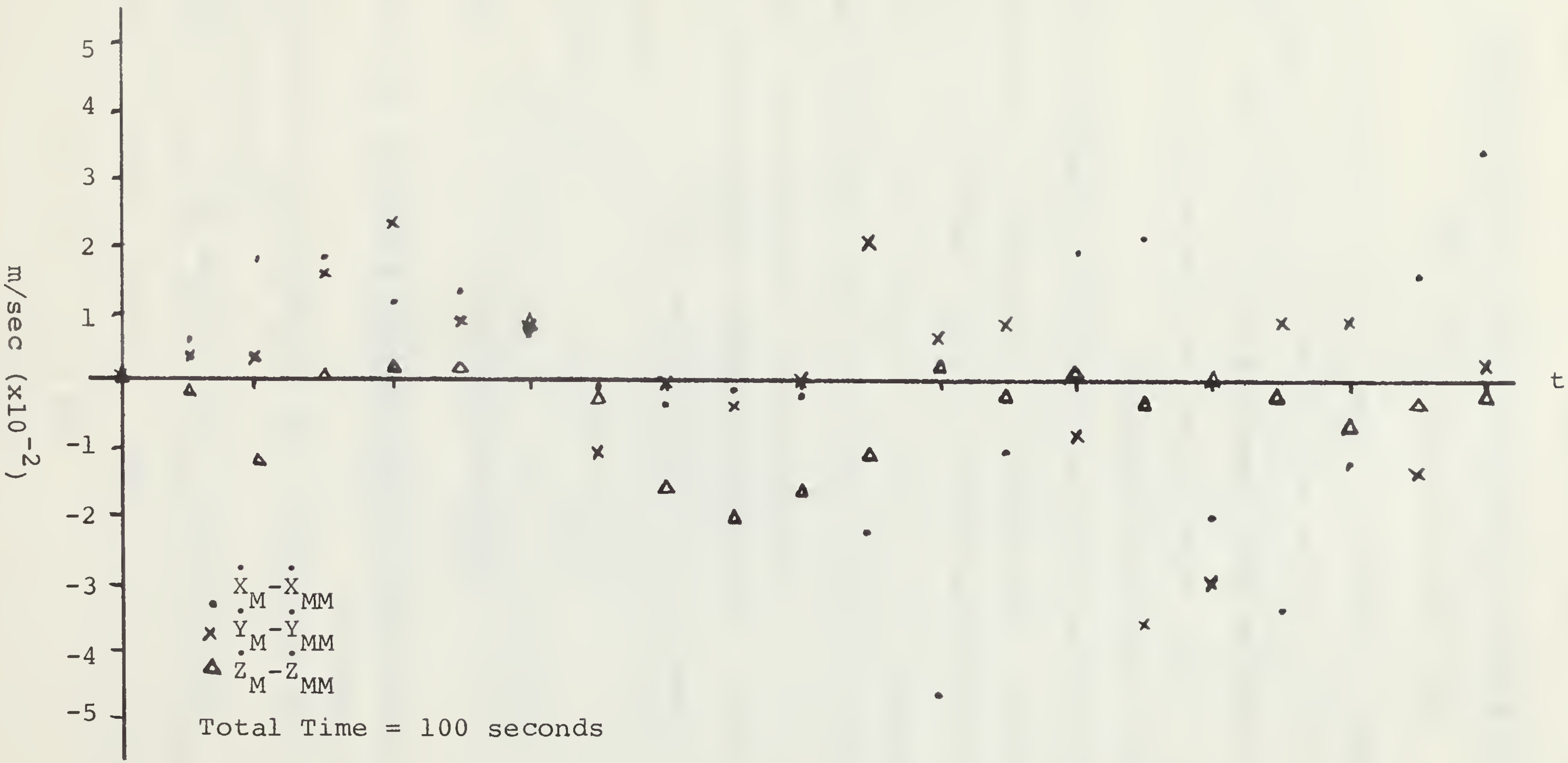

Figure 14. Missile Velocity Error vs. Flight Time

APPENDIX A

TARGET LOCATION BY SUCCESSIVE ANGULAR MEASUREMENTS FOR THE THREE-DIMENSIONAL CASE

In chapter II there was given a brief description of locating a target in the two-dimensional plane by processing two sets of angular measurements and two associated aircraft positions. In this appendix the two-dimensional case will be expanded to the third dimension.

Figure 15 shows the three-dimensional geometry involved in this problem. This figure is then shown in two plane views: the XY plane projection in Fig. 16 and the RZ plane projection of Fig. 17.

From Fig. 16 the two locations of the launching aircraft are marked as X_1, Y_1 and X_2, Y_2 with the associated azimuth angles θ_1 and θ_2. Using the geometric relationship for tangent, the tangents of the angles shown are

$$\frac{X_t - X_1}{Y_t - Y_1} = \tan \theta_1 \qquad \text{(A-1)}$$

$$\frac{X_t - X_2}{Y_t - Y_2} = \tan \theta_2 \quad . \qquad \text{(A-2)}$$

Treating the equations as simultaneous equations and setting them up for solution,

$$X_t - Y_t \tan \theta_1 = X_1 - Y_1 \tan \theta_1 \qquad \text{(A-3)}$$

$$X_t - Y_t \tan \theta_2 = X_2 - Y_2 \tan \theta_2 \ . \qquad \text{(A-4)}$$

Letting $\tan\theta_1 = A$, $\tan\theta_2 = B$, $X_1 - Y_1 \tan\theta_1 = C$, and $X_2 - Y_2 \tan\theta_2 = D$ and substituting these values into equations A-3 and A-4 results in

$$X_t - AY_t = C \tag{A-5}$$

$$X_t - BY_t = D \ . \tag{A-6}$$

Rewriting these equations in matrix form gives

$$\begin{bmatrix} 1 & -A \\ 1 & -B \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix} \tag{A-7}$$

which has a solution of the form

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 1 & -A \\ 1 & -B \end{bmatrix}^{-1} \begin{bmatrix} C \\ D \end{bmatrix} \ . \tag{A-8}$$

The solution of this equation, arrived at by normal matrix algebra, is

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \frac{1}{A-B} \begin{bmatrix} -B & A \\ -1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} \ . \tag{A-9}$$

Making substitutions back into the solution, the X_t and Y_t of the target are

$$X_t = \frac{\tan\theta_1 \ (X_2 - Y_2 \tan\theta_2) - \tan\theta_2 \ (X_1 - Y_1 \tan\theta_1)}{\tan\theta_1 - \tan\theta_2} \tag{A-10}$$

$$Y_t = \frac{(X_2 - Y_2 \tan\theta_2) - (X_1 - Y_1 \tan\theta_1)}{\tan\theta_1 - \tan\theta_2} \tag{A-11}$$

The solution for locating the Z coordinate of the target will now be derived.

Figure 17 will be used to illustrate the derivation of the target Z location problem. Again using the tangent

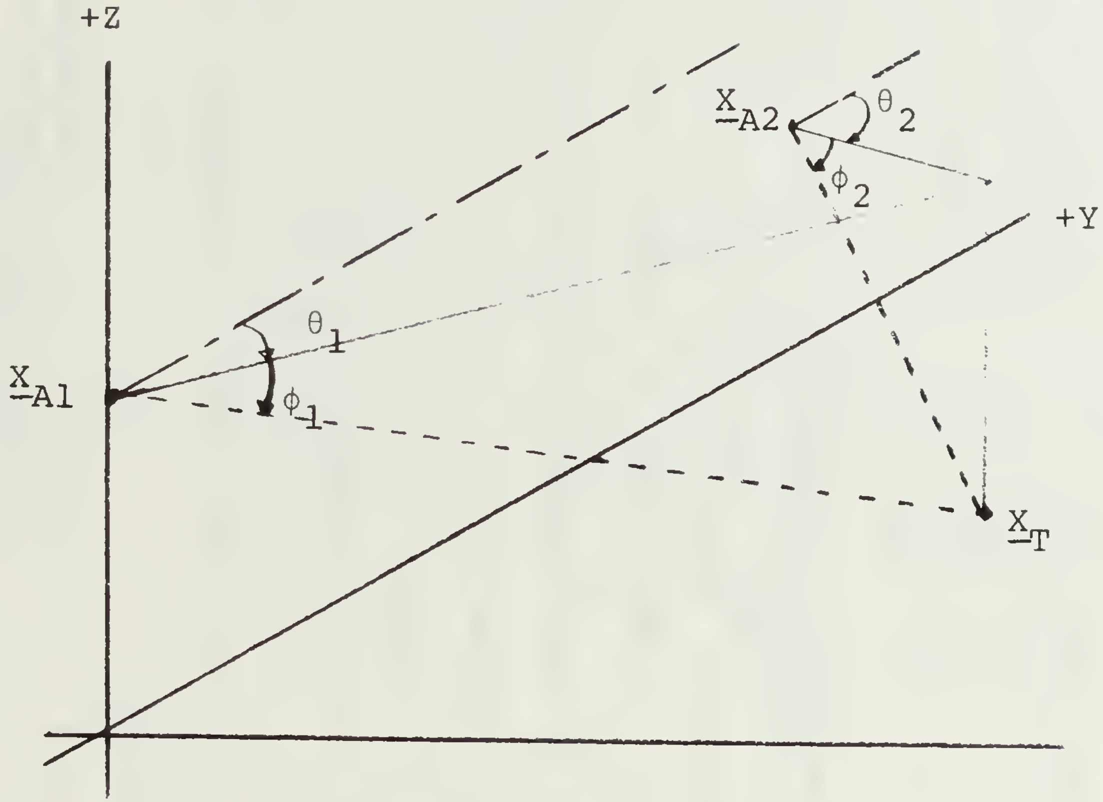

Figure 15. Three-Dimensional Location Geometry

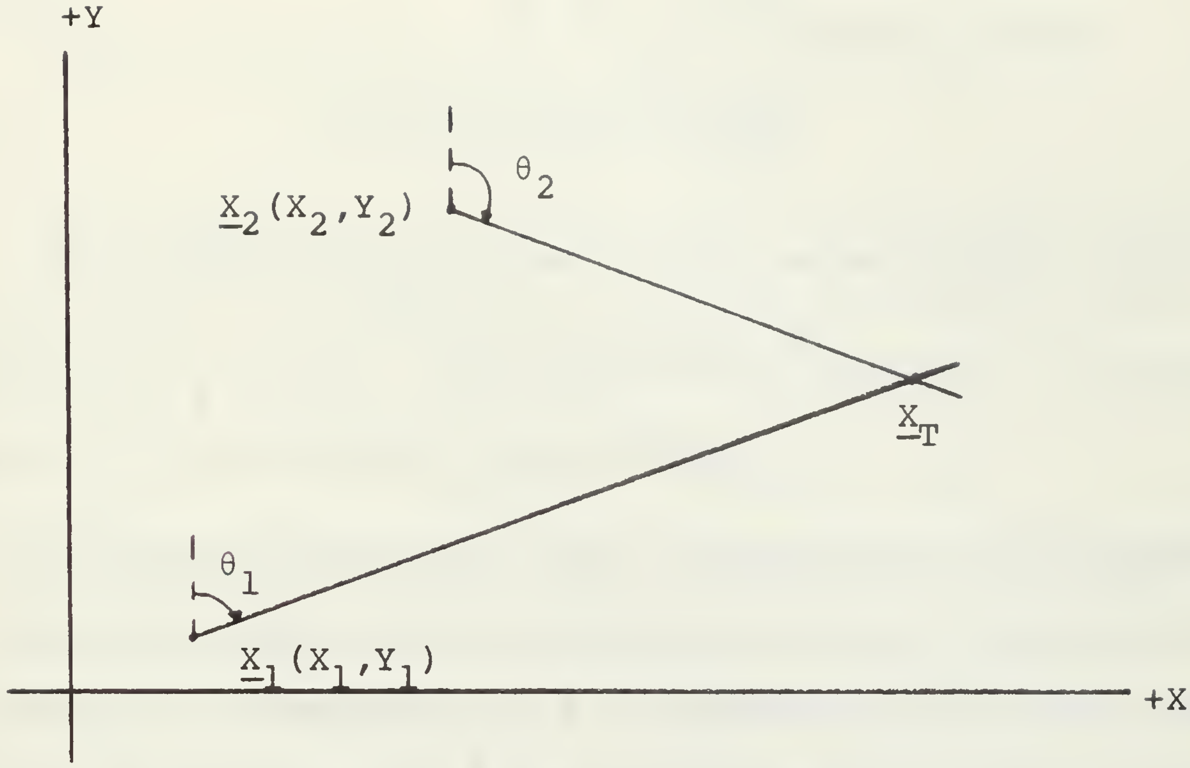

Figure 16. XY Plane View Location Geometry

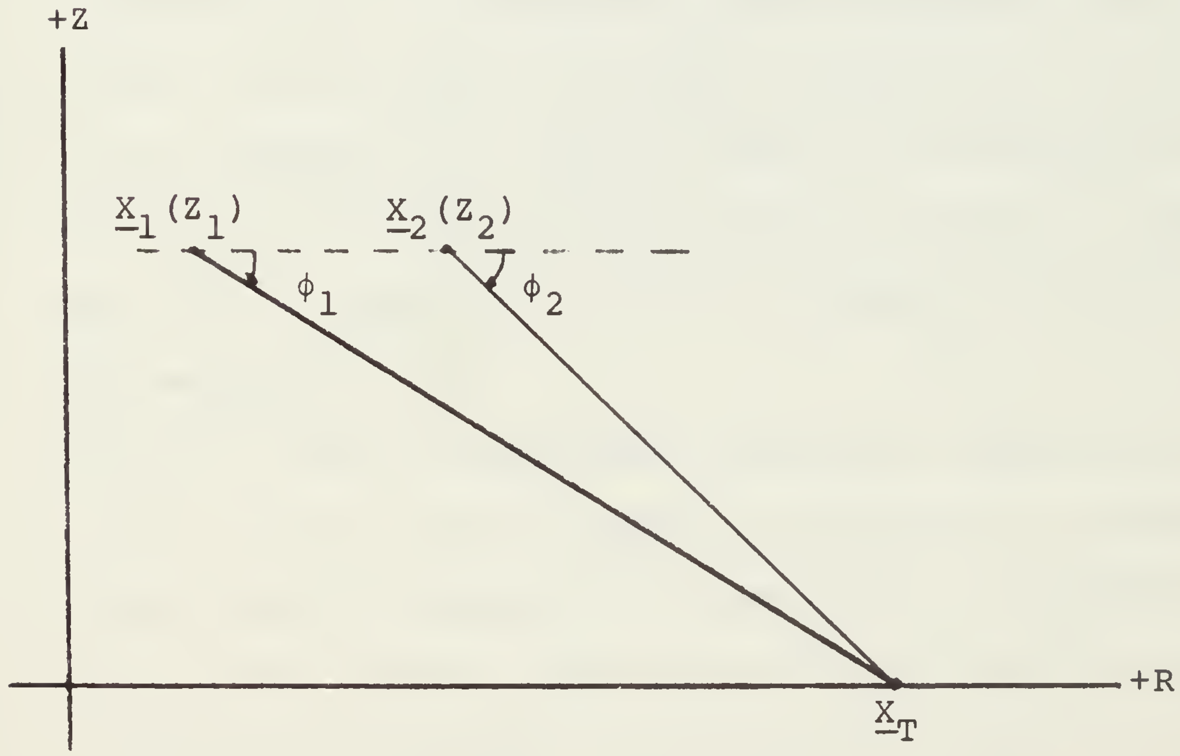

Figure 17. RZ Plane View Location Geometry

relationships for the angles involved, it can be seen that

$$\frac{Z_1 - Z_t}{D_1} = \tan \phi_1 \tag{A-12}$$

$$\frac{Z_2 - Z_t}{D_2} = \tan \phi_2 \quad . \tag{A-13}$$

From Fig. 16 the values for D_1 and D_2 can be seen to be

$$D_1 = ((X_t - X_1)^2 - (Y_t - Y_1)^2)^{\frac{1}{2}} \tag{A-14}$$

$$D_2 = ((X_t - X_2)^2 - (Y_t - Y_2)^2)^{\frac{1}{2}} \quad . \tag{A-15}$$

The fact is very evident from equations A-14 and A-15 that once the X and Y coordinates of the target have been found, any single measurement of the depression angle may be used along with the Z location of the aircraft at the time of the depression angle measurement to determine the Z coordinate of the target.

APPENDIX B

EFFECT OF A/R RATIO AND TARGET Z LOCATION ERROR ON RANGE MISS DISTANCE

Several references have been made in this paper to the A/R ratio and its importance in determining missile miss distances at the target. This appendix is devoted to a discussion of the importance of this effect.

A. DEFINITION OF A/R RATIO AND ITS EFFECT ON MISS DISTANCE

The A/R ratio, as defined for a parabola, is the ratio of the height of the parabola at its apex to the distance from a point under the apex to one of the terminal points of the parabola. This ratio is illustrated in Fig. 18. It can be seen from the figure that the larger the A/R ratio, the more nearly perpendicular the terminus of the parabola at its intersection with the axis. The more perpendicular the terminus of the parabola, the less the point of intersection with the target plane will be affected by an error in locating the target plane. Figures 19 and 20 illustrate the relationship just described.

In Fig. 19, which illustrates a small A/R ratio, it can be seen that for an error in target Z location, ΔZ, a relatively large error in range, ΔR, is generated. The alternate and desirable case of the large A/R ratio is shown in Fig. 20. For the same error in target Z location as in Fig. 19, the range error, ΔR, is much smaller than in the small A/R ratio example.

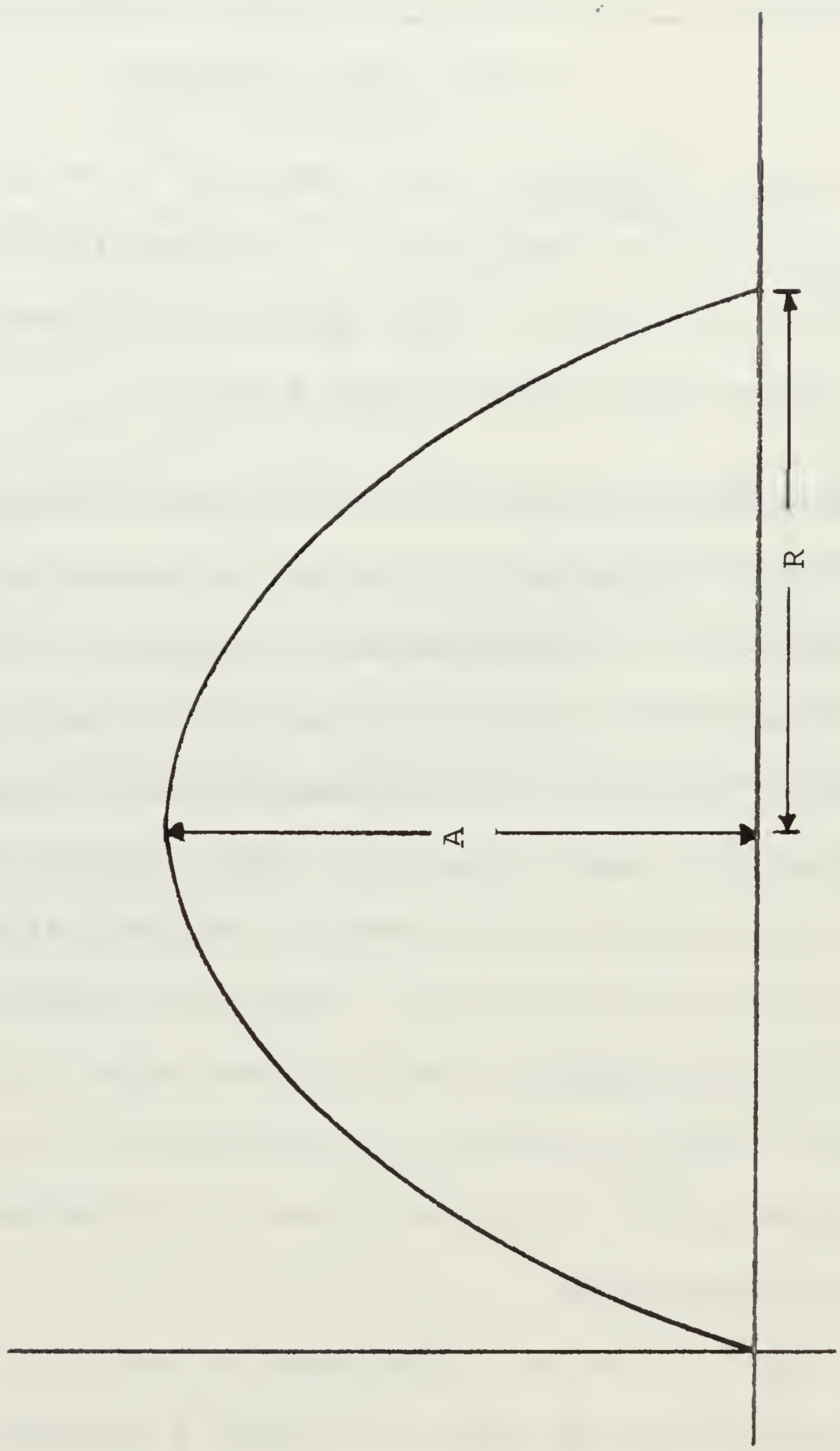

Figure 18. Illustration of A/R Ratio

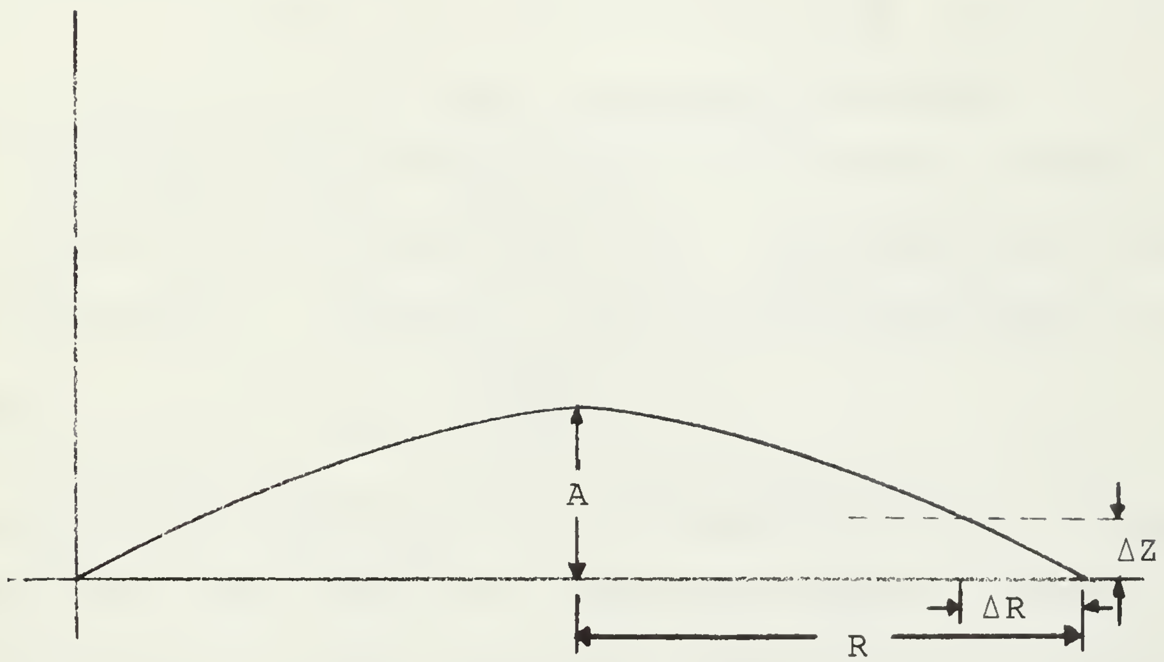

Figure 19. Small A/R Ratio

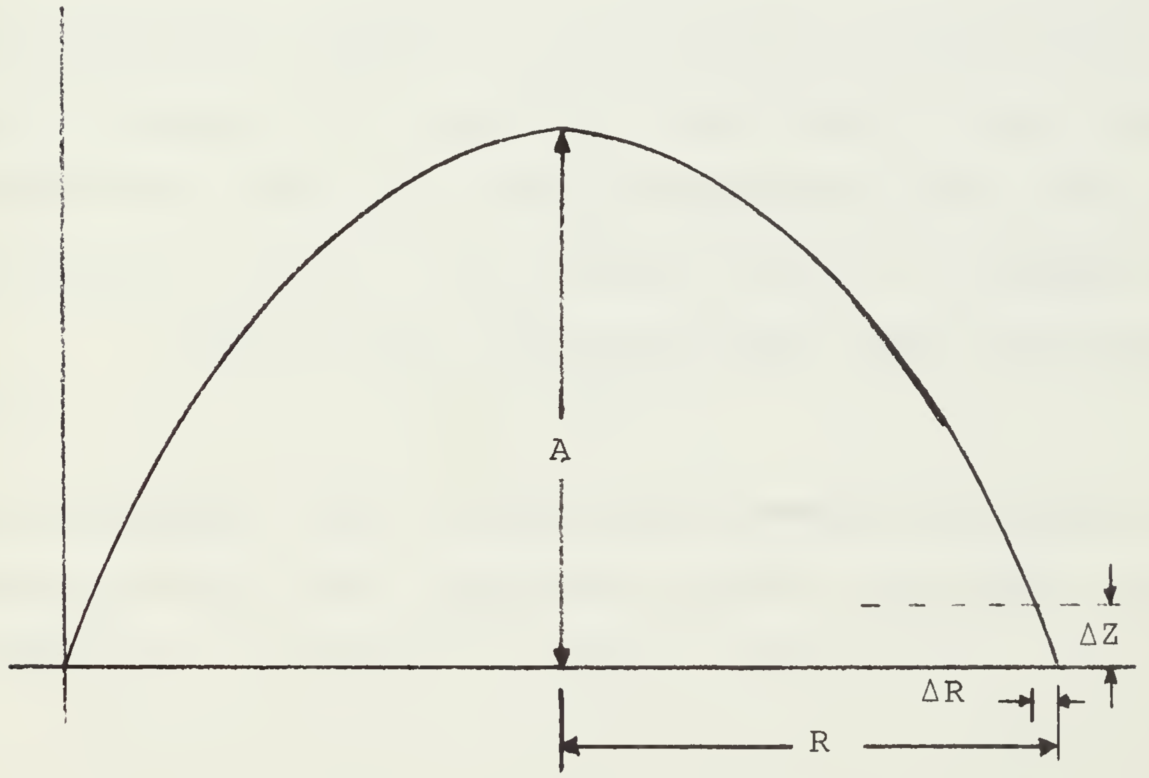

Figure 20. Large A/R Ratio

B. CURVES RELATING MISS DISTANCE AND A/R RATIO AS A FUNCTION OF TARGET ΔZ.

The computation of a family of curves relating miss distances and A/R ratio as a function of target ΔZ is straightforward. Assuming a ballistic, drag-free mass, the time for that mass to fall the distance A under the influence of gravity was determined. This was done for the initial condition of the mass having a zero Z velocity. In equation form

$$t_k = \frac{-\sqrt{-2\ddot{Z}A}}{\ddot{Z}} \tag{B-1}$$

where $\ddot{Z} = -9.8 \text{ m/sec}^2$ and A is the initial Z of the mass with the target Z assumed to be zero. By using this time and the knowledge of the distance R from the apex of the parabola to the target, the initial R velocity was computed as

$$V_r = \frac{R}{t_i} \quad . \tag{B-2}$$

This velocity was then set as constant throughout the current run in which the Z location of the target was varied away from the perfect zero location for which the R velocity was computed. For each value of ΔZ, a time to fall, t_f, was computed and then substituted into the equation

$$R = V_r t_f \quad . \tag{B-3}$$

The difference between this distance thus computed and the R to the target was the miss distance. The series of computations was repeated for a number of different A/R ratios and ΔZ's.

The results were then plotted with A/R ratio on the X scale and miss distance on the Y scale. The family of curves

of Fig. 21 was generated by connecting the miss distances at the different A/R ratios which were the result of the same ΔZ.

The values used in this problem were

$R = 5000$ meters (constant)

A's such that A/R ratios of .1, .2, .3, .4, .5, .6, .7, .8, .9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 were generated.

ΔZ's used were 1, 2, 5, 10, 20, 50, 100, 200, and 500 meters.

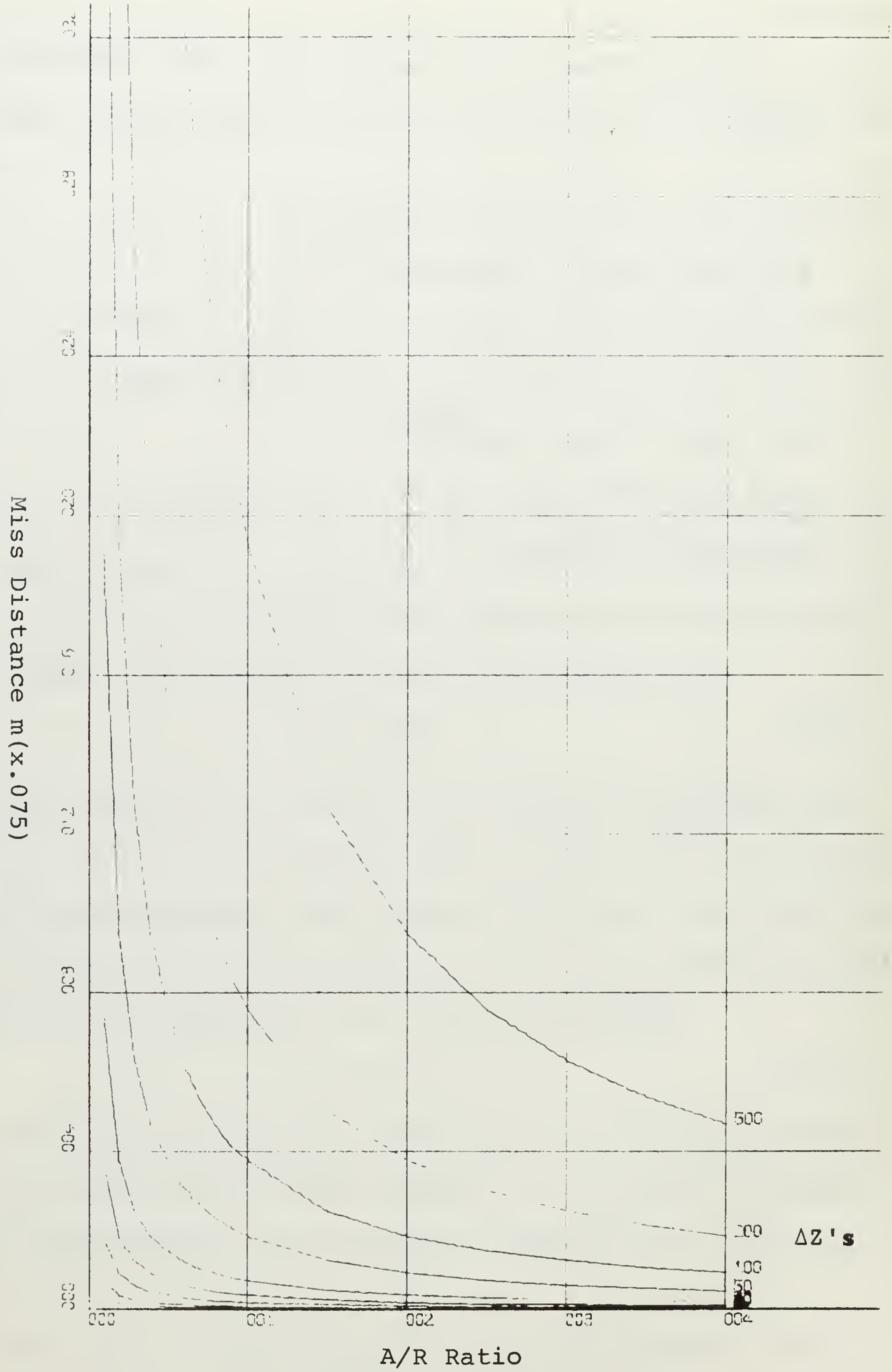

Figure 21. Range Miss Distance as a Function of ΔZ and A/R Ratio

APPENDIX C

DERIVATION OF THE KALMAN FILTER FOR THE MISSILE-TRACKING RADAR[2]

In chapter II the Kalman filter for the system radar was discussed. This appendix gives the assumptions which were used in the derivation of the filter.

The filter was designed with the assumption that the missile-tracking radar provides measurements of range, range rate, azimuth angle, and depression angle to the missile relative to the aircraft. Another assumption was that the measurements were contaminated by additive noise whose statistics were known. It was also assumed that the missile was not subjected to any random perturbations. Based on these assumptions, some of the matrices of equations C-1, C-3, and C-5 may be defined.

$$G_k = P_{k|k-1}H^t(HP_{k|k-1}H^t+R)^{-1} \tag{C-1}$$

$$P_{k|k} = (I-G_kH)P_{k|k-1} \tag{C-2}$$

$$P_{k+1|k} = \Phi P_{k|k}\Phi^t + Q \tag{C-3}$$

$$\hat{\underline{X}}_{k|k} = \hat{\underline{X}}_{k|k-1} + G_k(\underline{Z}_k-H\hat{\underline{X}}_{k|k-1}) \tag{C-4}$$

$$\hat{X}_{k|k-1} = \Phi\hat{\underline{X}}_{k-1|k-1} \tag{C-5}$$ [1]

The Q matrices are identically zero since it was assumed that the missile was subject to no random perturbations. The

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[1] The symbols used in the equations C-1 through C-5 and elsewhere in this appendix differ somewhat from the symbology used in R. C. K. Lee. [Ref. 2].

R matrices are not zero since they convey to the filter information regarding the variance of the additive noise in the measured values.

The Φ matrices are the same for all the filters since the same dynamics (constant velocity) were assumed for all the filters. Equation C-6 gives these Φ matrices as

$$\Phi_{rg} = \Phi_{az} = \Phi_{dp} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} . \qquad \text{(C-6)}$$

The H matrices in equations C-1, C-2, and C-4 are measurement matrices which determine which variables are measurable by the filters. In the range filter H_{rg} is a 2 X 2 identity matrix. The H_{az} and H_{dp} are 1 X 2 matrices with the (1,1) elements being unity.

It should be noted that equations C-1, C-2, and C-3 are data-independent as long as the matrices involved in computing them are data-independent and if the H, R, Φ, and Q matrices are constants. If this is true, then the gain schedule can be computed offline from the actual simulation of the problem or operational environment and simply inserted into the computer memory for use at the proper time. With the precomputed gain schedule in use, the implementation of the filter to process data in real-time is a relatively easy matter and does not contribute greatly to the computational load on the computer.

An important consideration in designing any filter is the expected value of the measured parameters which will be used to initialize the filter. This is especially important when choosing values for the initial covariance matrices, $P_{1|0}$.

As these filters were to be employed in conjunction with a missile-tracking radar, the assumptions were made that the initial range to the missile and range rate of the missile would be unknown and that the initial angles to the missile would also be unknown. These assumptions were reflected in the covariance matrix initializations when the gain schedule was computed. The values used are given for the range and range rate filter in equation C-7 and the angular filters in equation C-8.

$$P_{1|0} = \begin{bmatrix} 10000.\ m^2 & 0 \\ 0 & 1000.\ m^2/sec^2 \end{bmatrix} \qquad \text{(C-7)}$$

$$P_{1|0} = \begin{bmatrix} 10\ rad^2 & 0 \\ 0 & 10\ rad^2/sec^2 \end{bmatrix} \qquad \text{(C-8)}$$

The values used for the R matrices in the gain schedule computation were

$$R_{rg} = \begin{bmatrix} 100\ m^2 & 0 \\ 0 & 10\ m^2/sec^2 \end{bmatrix} \qquad \text{(C-9)}$$

$$R_{az} = R_{dp} = .0004\ rad^2 \quad . \qquad \text{(C-10)}$$

By using very large initial covariance matrices and setting the initial estimates of the state vectors to zero, the gain element (1,1) for the first time increment will be very close to unity, thus causing the first measurement of the particular state variable to become the first estimate of that variable. As time passes, however, the incoming measurement is weighted less and less as the gain goes down, i.e., the filter becomes more confident in the accuracy of its estimates.

However, this reduction of gain as a function of time can sometimes be self-defeating. In the case in which this filter was employed, the reduction of the gain with time could be disastrous. Some corrective action had to be taken to prevent the gains from approaching zero and, as a result, having the filter ignore new data. Otherwise, as the missile maneuvered, the filters would have very quickly begun to produce very bad estimates of the radar state vector.

Thus, the effect which is desired is one which will not allow the gains to approach zero. There are two actions which will prevent this from occurring: letting the Q matrices be non-zero, or truncating the gain schedule after some length of time kT. Since the Q matrices have already been considered to be zero, there remains gain-schedule truncation. By truncating the gain schedule the filters continue to accept incoming measurements as containing significant information, and the amount of core storage in the computer is reduced greatly.

The gain schedules in this particular simulation were truncated in such a manner that the previous estimate produced by the filters were weighted by a factor of approximately 0.8 and the incoming measurements were weighted by a factor of approximately 0.2.

APPENDIX D

DERIVATION OF THE SOLUTION TO THE IMPACT PREDICTION EQUATIONS

This appendix discusses the derivation of the impact prediction equations used for the missile impact point prediction. Two methods are discussed: one in which the drag force is proportional to the velocity and a second in which the drag force is proportional to the velocity squared. The first was used in the simulation program.

In order to derive closed-form solutions to the equations of motion for the missile, it was assumed that the drag coefficient was constant over the prediction interval.

This assumption will introduce errors into the predictions. However, this error should be reduced the closer the missile approaches to its terminal point. Near the terminal point of the flight the greatest error will be in the predicted time until impact since the Z acceleration will be the greatest at that time. This should not greatly affect the X and Y prediction since the X and Y accelerations will tend to be very small at that point.

The method used to solve the equations is to first solve the Z motion equation for the time at which the missile Z is equal to the target Z. The value of time thus found, which is the predicted remaining flight time of the missile, is then used with the current X and Y positions and velocities to compute the predicted impact point.

A. METHOD I - DRAG PROPORTIONAL TO VELOCITY

In method I the drag force on the missile is considered to be proportional to the velocity of the missile. The equations for this case are linear differential equations.

The equations of motion for method I are

$$m\ddot{X} = -K_1\dot{X} \quad \text{(D-1)}$$

$$m\ddot{Y} = -K_2\dot{Y} \quad \text{(D-2)}$$

$$m\ddot{Z} = -K_3\dot{Z} - mG \quad \text{(D-3)}$$

where

$$K_1 = \rho|\dot{X}|A_r \quad \text{(D-4)}$$

$$K_2 = \rho|\dot{Y}|A_r \quad \text{(D-5)}$$

$$K_3 = \rho|\dot{Z}|A_r \quad \text{(D-6)}$$

in which the $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ are the current values of the velocities of the missile, ρ is the atmospheric density, and A_r is a reference area of .114 meter^2.

The solution for the Z motion equation D-3 has the form

$$0 = -Ae^{-\alpha t} - Bt + C \quad \text{(D-7)}$$

where

$$A = \frac{m\dot{Z}_m(0)}{K_3} + \frac{m^2G}{K_3^{\,2}} \quad \text{(D-8)}$$

$$B = \frac{mG}{K_3} \quad \text{(D-9)}$$

$$C = A + Z_m(0) - Z_t \quad \text{(D-10)}$$

$$\alpha = \frac{K_3}{m} \quad . \quad \text{(D-11)}$$

The problem now becomes the solution of the time-domain equation of the Z motion for the time at which the missile Z is equal to the target Z. The C coefficient in equation D-10 is set to reflect this desired equality already.

The method chosen to solve this equation is the Newton-Raphson method. Upon commencement of the iterations all of the required coefficients were computed using the current values of the positions and velocities. The termination test constant used to terminate the iterative process was 0.1 second. Whenever one value of time had changed from the previous value of time by an amount less than or equal to the test increment, the iterative process was terminated. The value of 0.1 seconds was chosen for the test value since regardless of the sample interval given to the simulation program the missile model was advanced in increments of 0.1 seconds. Thus, whenever the time was less than or equal to the minimum increment that the missile could be advanced, the iteration was stopped.

The value of time which was returned from the Newton-Raphson iteration was then substituted into the equations for X and Y impact point prediction. These equations are

$$X_p = X_m(0) + \frac{m\dot{X}_m(0)}{K_1}(1.0 - e^{-\beta t}) \qquad \text{(D-12)}$$

$$Y_p = Y_m(0) + \frac{m\dot{Y}_m(0)}{K_2}(1.0 - e^{-\gamma t}) \qquad \text{(D-13)}$$

where

$$\beta = \frac{K_1}{m} \qquad \text{(D-14)}$$

$$\gamma = \frac{K_2}{m} \quad . \tag{D-15}$$

These are the equations for method I prediction. The method II equations are more complicated to solve since they are non-linear.

B. METHOD II - DRAG PROPORTIONAL TO VELOCITY SQUARED

In this derivation of impact prediction equations the drag force on the missile is taken to be proportional to the square of the missile velocity. Again, the drag coefficients are computed at the beginning of the solution and considered constant throughout.

The equations of motion are

$$m\ddot{X} = -K_1\dot{X}^2 \tag{D-16}$$

$$m\ddot{Y} = -K_2\dot{Y}^2 \tag{D-17}$$

$$m\ddot{Z} = -K_3\dot{Z}^2 - mG \quad . \tag{D-18}$$

The steps of this solution are the same as used in the previous method: 1) solve the Z motion equation for Z as a function of time, 2) solve for the time when missile and target Z are equal, and 3) use the time thus found to compute X_p and Y_p from the prediction equations.

First solve for Z as a function of time, writing equation D-18 as

$$\frac{dy}{dt} = -(Cy^2+G) \tag{D-19}$$

where

$$C = \frac{K_3}{m} \tag{D-20}$$

and

$$y = \dot{Z} \quad . \tag{D-21}$$

Separating variables and integrating,

$$-\frac{1}{\sqrt{CG}}\tan^{-1}(y\sqrt{C/G}) + D_1 = t \ . \tag{D-22}$$

At $t = 0$, $y = \dot{Z}_m(0)$. Therefore

$$D_1 = \frac{1}{\sqrt{CG}}\tan^{-1}(\dot{Z}_m(0)\sqrt{C/G}) \quad . \tag{D-23}$$

Solving for y and substituting for D_1,

$$y - \sqrt{G/C}\tan(D_1\sqrt{CG}) = -\sqrt{G/C}\tan(t\sqrt{CG}) \tag{D-24}$$

or

$$y = -\sqrt{G/C}\tan(t\sqrt{CG}) + \dot{Z}_m(0) \quad . \tag{D-25}$$

Substituting $y = \frac{dZ}{dt}$ and letting $\dot{Z}_m(0)$ be constant,

$$dZ = [-\sqrt{G/C}\tan(t\sqrt{CG}) + \dot{Z}_m(0)]dt \tag{D-26}$$

and

$$Z = -\sqrt{G/C}\frac{1}{\sqrt{CG}}\log_e\cos(t\sqrt{CG}) + \dot{Z}_m(0)t + D_2 \ . \tag{D-27}$$

At $t = 0$, $Z = Z_m(0)$. Therefore $D_2 = Z_m(0)$ and

$$Z(t) = -\frac{1}{C}\log_e\cos(t\sqrt{CG}) + Z_m(0)t + Z_m(0) \tag{D-28}$$

where $\dot{Z}_m(0)$ and $Z_m(0)$ refer to the Z velocity and position of the missile at the time the prediction is made.

Solving for the time when Z(t) is equal to the target elevation is done by substituting $Z(t) = Z_t$ and using the Newton-Raphson method.

The solution of the X and Y motion equations for impact point will be shown for the X equation only and the Y equation will be written by inspection.

$$m\ddot{X} = -K_1\dot{X}^2 \tag{D-29}$$

$$\frac{dy}{dt} = -Cy^2 \tag{D-30}$$

where $y = \dot{X}$ and $C = \frac{K_1}{m}$. Separating variables and integrating

$$\frac{1}{Cy} + D_1 = t \quad . \tag{D-31}$$

At $t = 0$, $y = \dot{X}_m(0)$. Therefore

$$D_1 = -\frac{1}{C\dot{X}_m(0)} \quad . \tag{D-32}$$

Substituting $y = \frac{dX}{dt}$ and letting $\dot{X}_m(0)$ be constant

$$\frac{dt}{dX} = Ct - CD_1 \quad . \tag{D-33}$$

Separating variables and integrating

$$X = \frac{1}{C} \log_e(Ct - \frac{1}{\dot{X}_m(0)}) + D_2 \quad . \tag{D-34}$$

At $t = 0$, $X = X_m(0)$. Therefore

$$D_2 = X_m(0) - \frac{1}{C} \log_e(- \frac{1}{\dot{X}_m(0)}) \tag{D-35}$$

and

$$X(t) = \frac{1}{C} \log_e(Ct - \frac{1}{\dot{X}_m(0)}) - \frac{1}{C} \log_e(- \frac{1}{\dot{X}_m(0)}) + X_m(0) \tag{D-36}$$

$$X(t) = \frac{1}{C} \log_e(Ct - \frac{1}{\dot{X}_m(0)})(-\dot{X}_m(0)) + X_m(0) \tag{D-37}$$

or

$$X(t) = \frac{m}{K_1} \log_e[1 - \frac{K_1 t\dot{X}_m(0)}{m}] + X_m(0) \quad . \tag{D-38}$$

For this method the prediction equations are

$$X_p = \frac{m}{K_1} \log_e[1 - \frac{K_1 t\dot{X}_m(0)}{m}] + X_m(0) \tag{D-39}$$

and

$$Y_p = \frac{m}{K_2} \log_e[1 - \frac{K_2 t\dot{Y}_m(0)}{m}] + Y_m(0) \quad . \tag{D-40}$$

APPENDIX E

REFERENCE SYSTEMS FOR THE PROBLEM

The reference systems for the problem are based on the position of the launch point of the aircraft involved. The inertial navigator of the aircraft is initialized when the aircraft is launched. All other points in the problem are also referred to this initialization point of the aircraft even though they may not be measured with respect to that point at first. The measured missile state vector is an example of this type in that it is first referred to the mother aircraft and then converted to a vector which is referred to the initialization point.

The cartesian reference is a standard three-dimensional, right-hand reference system. Whenever a reference is shifted, the new frame is also a three-dimensional, right-hand system. Figure 22 illustrates the various reference points and systems.

Azimuth angle is defined as the angle between true north, positive clockwise to the line-of-sight from the aircraft to the item of interest. In order to define the depression angle, it is first necessary to define the normal plane. The normal plane is defined as being a plane normal to a line passing from the aircraft to the center of the earth. Depression angle is defined as the angle between the normal plane and the line-of-sight. This LOS must be contained in a plane which is perpendicular to the normal plane. These angles are shown in Fig. 23 and 24 respectively.

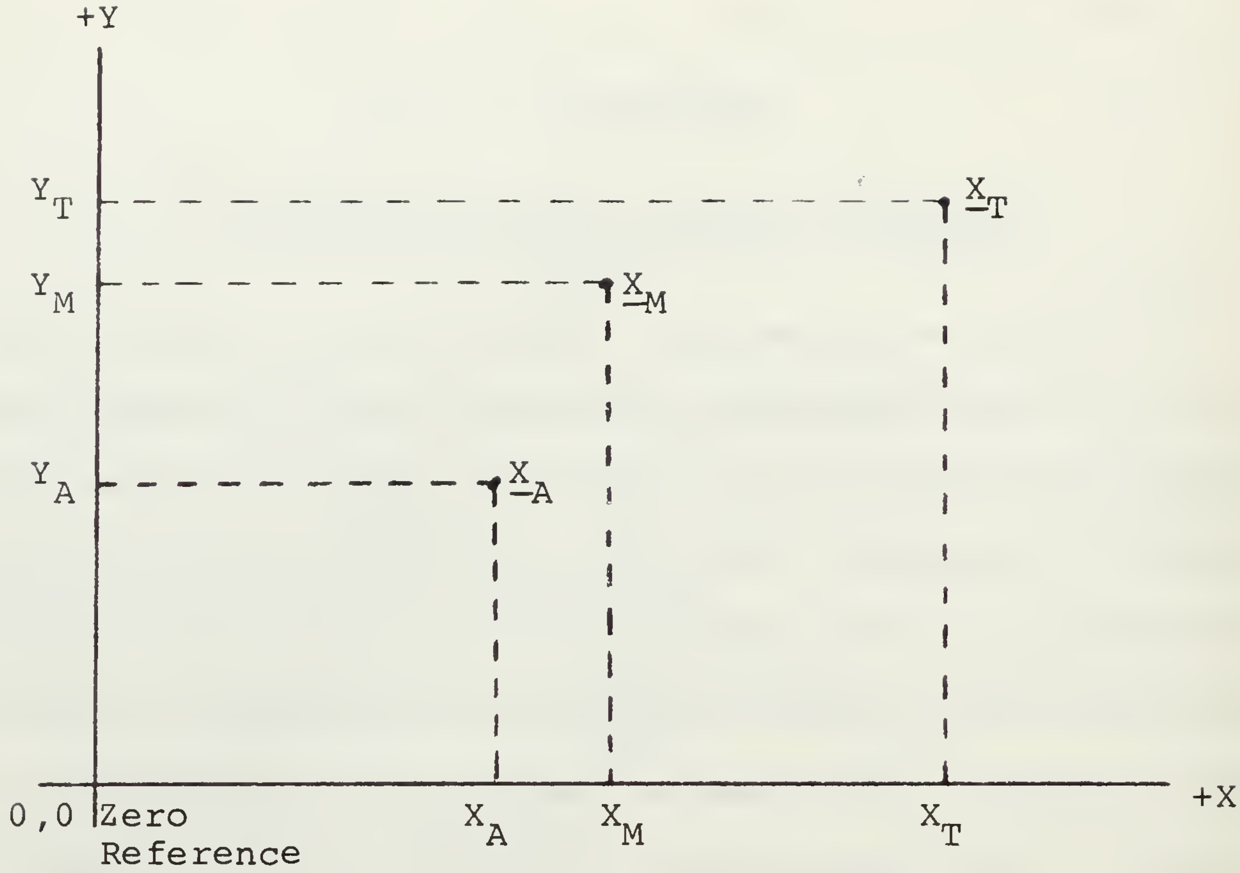


Figure 22. Problem Cartesian Reference Frame

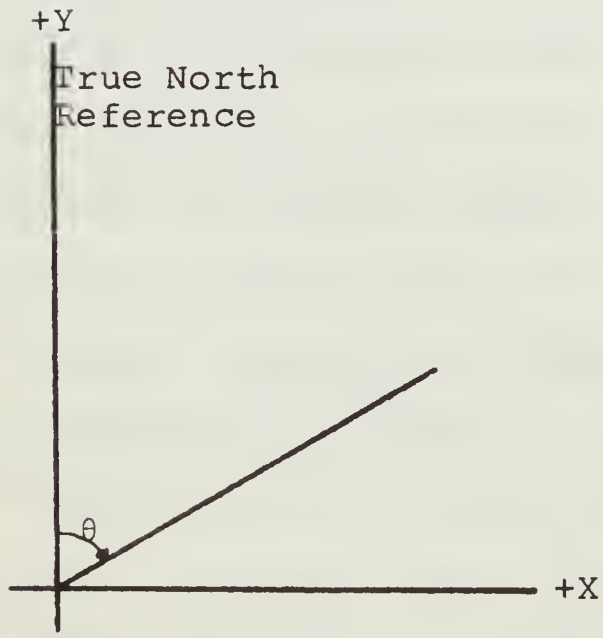


Figure 23. Definition of Azimuth Angle

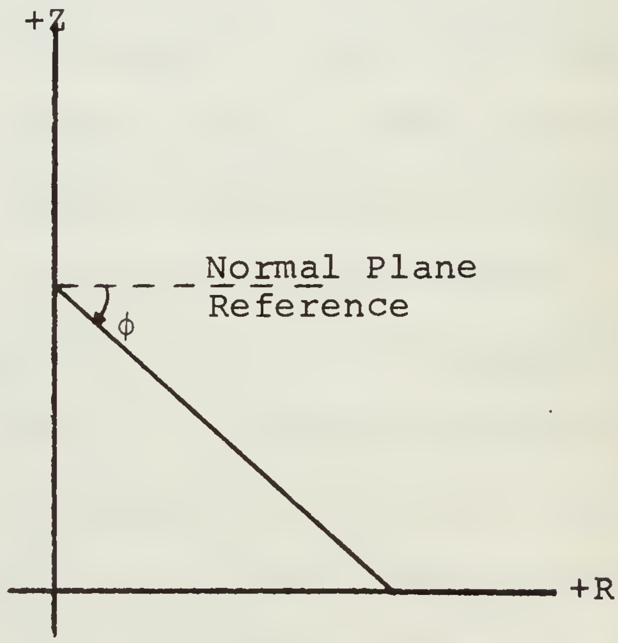


Figure 24. Definition of Depression Angle

APPENDIX F

THE MISSILE MODEL

The missile model was relatively honest in its aerodynamic performance although it was not a true missile. The maximum yaw rate of the missile was limited to allow radial acceleration loads not to exceed ten G's. This was thought to be representative of current capabilities.

The missile was powered by a rocket motor of 3000-lbs. thrust with a burn time of 30 seconds. After the burnout the missile glided for the remainder of its flight.

The model was so designed that the mass and moment of inertia changed as the rocket motor burned. The total propellant weight was approximately 163.3 kg. The reduced mass and moment of inertia were then used for the remainder of the flight. A free body diagram of the missile appears in Fig. 25. The equations of motion are

$$m\ddot{R} = -D\cos\gamma - L\sin\gamma + T\cos\theta \tag{F-1}$$

$$m\ddot{Z} = -D\sin\gamma + L\cos\gamma + T\sin\theta - mG \tag{F-2}$$

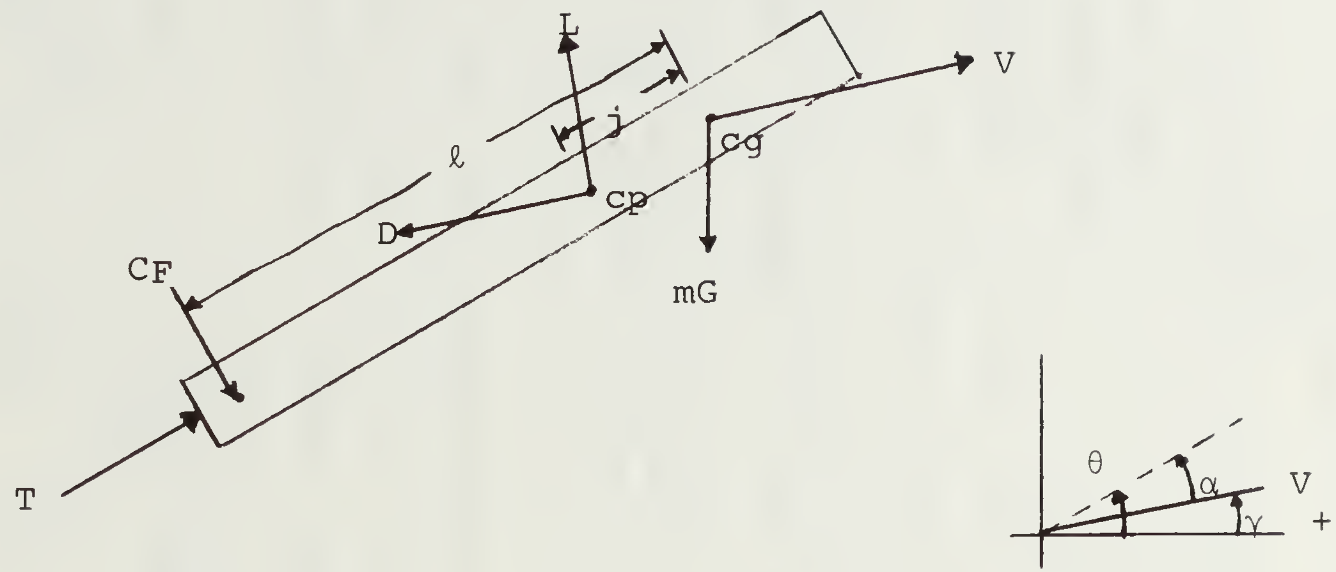
$$I_{yy}\ddot{\theta} = -jL - jD\sin\alpha + \ell CF - K_4 Q\dot{\theta} \tag{F-3}$$

where

$$Q = \rho V^2 \tag{F-4}$$

$$D = K_1 Q \tag{F-5}$$

$$L = K_5 Q\alpha \tag{F-6}$$



$$\theta = \alpha + \gamma \; .$$

$$\gamma = \tan^{-1}\left(\frac{Z}{\dot{R}}\right)$$

Figure 25. Missile Free-Body Diagram

and

T	(thrust)	13344. N
L	(lift)	N
D	(drag)	N
V	(velocity)	m/sec
CF	(control force)	N
γ	(velocity vector angle)	rad
α	(angle of attack)	rad
θ	(missile axis angle)	rad
$\dot{\theta}$	(axis rate)	rad/sec
$\ddot{\theta}$	(axis acceleration)	rad/sec^2
m	(mass)	453.6 kg (290.3 kg after burnout)
G	(gravity)	9.8 m/sec^2
ℓ	(control moment arm)	2. m
j	(center of pressure moment arm)	1. m
I_{yy}	(moment of inertia)	340.2 nt-m^2 (217.7 nt-m^2 after burnout)
ρ	(air density)	1.284 kg/m^3
K_1	(drag constant)	.00556 nt/(kg/m^3)(m/sec)^2
K_3	(control force constant)	.01 nt/(kg/m^3)(m/sec)^{2}rad
K_4	(damping constant)	.004 nt/(kg/m^3)(m/sec)^2(rad/sec)
K_5	(lift constant)	.008 nt/(kg/m^3)(m/sec)^{2}rad.

APPENDIX G

THE LAUNCHING AIRCRAFT MODEL

The launching aircraft model was a very simple model for the simulation. The aircraft was given an initial heading and velocity. It was then assumed to fly a straight line making no turns. The location of the aircraft was updated each sample period.

Although preliminary investigation had shown that the most critical component of noise in the entire system was that in the inertial navigation package, the inertial navigator was considered to be noiseless. The reasons for neglecting the noise were that 1) the primary purpose of this investigation was in the field of application of impact prediction to an operator-controlled, air-to-surface weapon system, and 2) no figures were available concerning sample-to-sample noise in inertial navigators.

APPENDIX H

MAN/SYSTEM INTERFACE

In any system with a human operator, it is important that information be presented concisely and in such a manner that it can best be used by the operator. The communication link in any such application is a two-way channel. The system sends information to the operator in one direction and the operator-generated control commands travel in the other.

The link to the operator carried two items of information. The first was the most recent prediction of impact point and the second was the predicted time until impact.

The first item, impact point prediction, was displayed to the operator on a display scope with the target assumed to be in the center. This presentation was limited in such a manner that the predicted impact point could never appear off-scale. Figures 26, 27, and 28 give examples of the limiting action which was employed. The display, when in a limited state, gave no information as to the error of the predicted impact point except the direction in which it was off-scale.

The display was generated such that the missile always approached the target from the bottom of the scope. This required a transformation of the axes of the display. This was necessary so that the motion of the control stick always drove the missile toward the target.

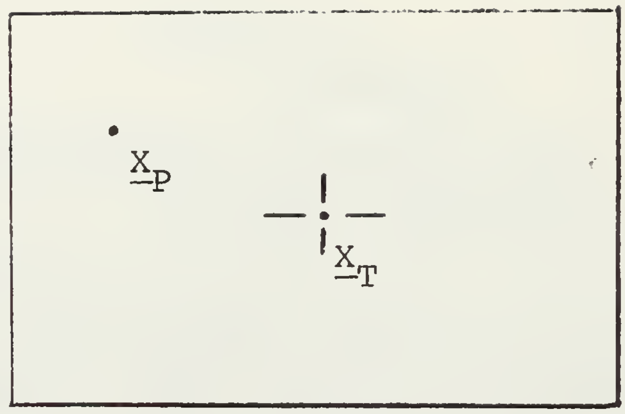


Figure 26. Impact Point Within Scale on Display

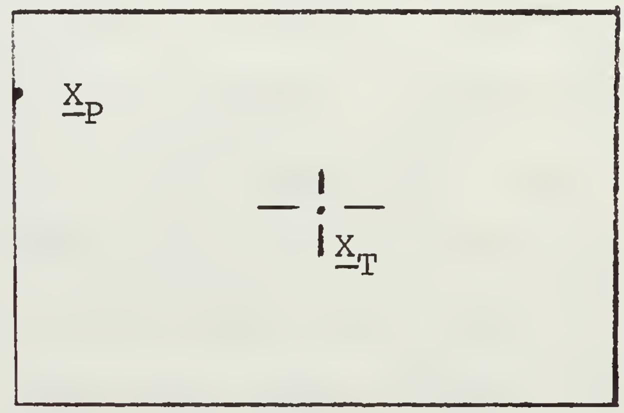


Figure 27. Impact Point Off-Scale Left on Display

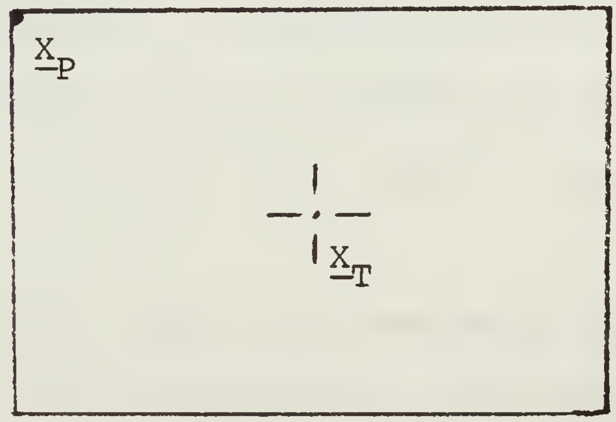


Figure 28. Impact Point Off-Scale Left, Off-Scale Vertically on Display

The transformation of axes consisted of both a translation and rotation. The translation was accomplished by generating a reference frame which had its origin located on the target. The predicted impact point was then referred to this origin and the axes rotated in such a manner as to make the Y axis of the rotated system coincide with the LOS from the current measured missile position to the target. This process is illustrated in Fig. 29 and 30.

It can be shown that the location of the predicted impact point in the translated and rotated reference frame (display reference system) is

$$\underline{X}_d = A\underline{X}_{df} \tag{H-1}$$

where

$$\underline{X}_d = \begin{bmatrix} X_d \\ Y_d \end{bmatrix} \tag{H-2}$$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{H-3}$$

$$\underline{X}_{df} = \begin{bmatrix} X_p - X_t \\ Y_p - Y_t \end{bmatrix} \tag{H-4}$$

and

$$\theta = \tan^{-1}((X_t - X_m)/(Y_t - Y_m)) \quad . \tag{H-5}$$

The second item of information presented to the operator was the predicted time remaining until impact. This value was computed once each sample period throughout the missile flight. It was generated on the digital computer, converted to an analog voltage, and displayed on a digital voltmeter. The display was scaled such that times were displayed only if

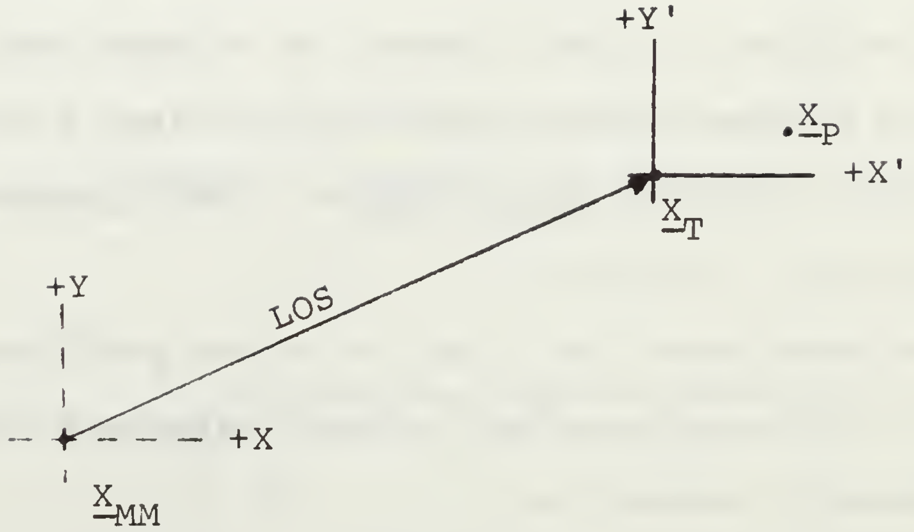


Figure 29. Translation of Axes for Display

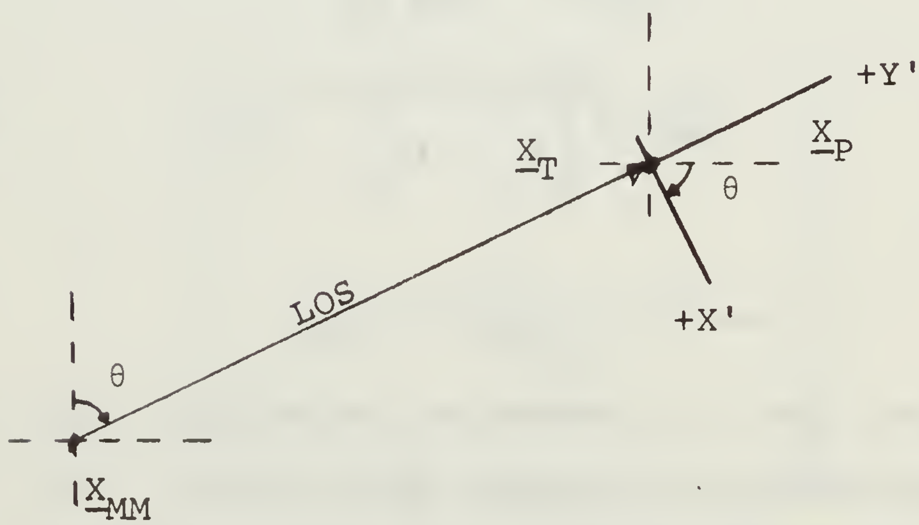


Figure 30. Rotation of Axes for Display

they were less than or equal to 100 seconds. Any value of time which was greater than 100 seconds was displayed as 100 seconds.

Commands from the operator to the system formed the other direction of the two-way communication link. These commands were generated by a control stick which the operator used to guide the missile. Full deflections on the stick corresponded to full deflections on the missile controls. The pitch control was scaled to provide 45 degrees of control movement on the missile when the stick was fully deflected. The yaw control provided a yaw rate of 20 degrees/second when fully deflected. This value was also subject to limiting by the missile dynamics.

APPENDIX I

A DISCUSSION OF THE SIMULATION PROGRAM

This appendix describing the program used in the simulation of this problem is included in the event that some other investigator would desire to continue research in this particular field. In it the author has tried to include all pertinent information concerning the routines and their functions and especially the programming "tricks" which were used in some instances to provide a particular function in the overall program.

The appendix is divided into four sections. The first section is a summary of the routines and the function which each performs. The second section lists the routines and the variable names within each routine. The method of inter-routine communication is the subject of the third section and the programming "tricks" used are discussed in a fourth section. A complete listing of the program and an analog patching diagram follow the appendix.

A. SUMMARY OF THE ROUTINES AND THEIR FUNCTIONS

1. Main Line Program

The main line program serves to perform two functions. The first is as a vehicle for input to the program and output from the program, and the second is as a calling and controlling sequence for the various routines which do the actual computation for the simulation. If operated in a segmented

mode of core allocation, the main line program is always core resident.

Several test functions are performed by the main line program also. It tests for the condition of problem termination. Problem termination occurs when the measured Z of the missile is equal to or less than the target Z. It also controls switching from the passive target location mode of the problem to the active mode in which the missile is fired and guided to the target.

2. Initialization Routines

a. Inertial Navigator Initialization

The subroutine INIT initializes the inertial navigation routine by setting the initial position of the aircraft including the altitude. It also initializes the heading and computes component velocities. The final items initialized by the routine are the elapsed time counter, missile flight time counter, and the problem time increment.

b. Missile Initialization

The subroutine MINIT initializes the missile model at the time of launch. All parameters of the missile model are set equal to the aircraft parameters at the time of launch. Also the routine gives initial values to the measured missile vector to enable other routines to function properly during the first second of the simulation until all routines are generating full outputs.

3. Aircraft Simulation Routine

The subroutine NAVPCK is the routine which provides aircraft state vector information for the problem. The routine provides a straight-line flight path for the aircraft at any initial heading from 0 to 359 degrees at any initial velocity. The routine receives flight path information in R/θ form and provides flight-path information in the three cartesian coordinates.

4. Passive Target Location

The subroutine PFIL simulates the action of the hybrid filter presented in section II-A-3. This location action is simulated by interpolating and normalizing values of variance provided for the hybrid method and generating noise which is added to the true target location. The final noisy location of the target is then used throughout the remainder of the problem.

5. Missile Model

The missile model routine is of sufficient importance that an entire previous appendix was devoted to it. The reader is referred to Appendix F for a discussion of the missile model.

6. Routines Involved in Missile Tracking

a. Radar Vector Generation

The subroutine REVCON is the routine which generates the true radar vector from the geometrical relationships between the true missile and the aircraft. This vector represents the true radar state before any noise is added and before quantization takes place.

b. Noise Addition and Quantization of Radar Vector

The subroutine RADAR2 takes the true radar vector produced by REVCON and performs two operations on it. First the routine adds the measurement noise associated with the particular measured state variable. Second, the routine quantizes the measurements into the discrete form which would be provided by a digitized radar output unit.

c. Radar Vector Filtering

The subroutine AFIL provides the filtering of the noisy, quantized radar vector to produce a radar vector whose variance is less than that of the noisy vector. The subroutine uses a schedule of precomputed filter gains and a switch which determines the number of samples after which the gain schedule is truncated. This value can be from one to thirty seconds assuming gains computed for a sample time of one second.

d. Spherical to Cartesian Coordinate Conversion

The subroutine CORDCN processes the filtered radar vector spherical values to produce a measured missile vector composed of missile position and velocities relative to the true problem reference. The subroutine first produces a cartesian vector relative to the aircraft. The aircraft component positions and velocities are then added to the relative components to produce the final measured missile vector.

7. Initial Missile Control Routine

The subroutine SHOOT controls the flight of the missile during the boosted phase (first thirty seconds) of its flight. The routine attempts to drive the missile velocity vector to assume an up angle of 47.5 degrees and to turn the missile velocity vector projection on the XY plane to coincide with the LOS from the missile to the target.

8. Impact Prediction and Display

a. Impact Point Prediction

The subroutine PREDCT develops the X and Y impact point prediction values. This routine employs the subroutine NR to assist in this task.

b. Time-Until-Impact Solution

The subroutine NR solves for the time to go until impact for the missile by using the Newton-Raphson iterative method of solution for differential equations.

c. Command Generation

The subroutine COMGEN converts the outputs of the control stick to values which can drive the missile model. This routine uses the SDS 9300 routines for analog-to-digital conversion. Full-scale values of pitch correspond to plus or minus 45 degrees of control deflection and full-scale values of yaw correspond to turn rates of plus or minus 20 degrees/second.

9. Prediction Display and Statistics Routines

a. Display Generation

The subroutine DISGEN converts the predicted impact points into values which can be used to drive the

situation display. This routine performs the axis rotation and translation necessary to generate a useful display. The limiting of the display range is also performed.

b. Minimal Statistics Routine

The subroutine MISS provides minimal statistics about the success of the run at the time of flight termination. This routine provides the X and Y miss distances, impact time of the missile, and total miss distance. More complete statistics are generated by a separate program which analyzes data which is written on a storage medium during the execution of the program. The separate analysis routine provides such results as missile flight profile, control applied vs. time, missile location errors vs. time, and others.

B. ROUTINE SYMBOLS AND THEIR MEANINGS

1. Main Line Program

Symbol	Meaning
GBQ/KBQ	Time for active filter gain truncation.
RUNS/RNS	Number of runs for the program.
NP	Switch used to initialize AFIL.
SMU, ARNG, ARNGD, AAZ, ADP	Mean of noise and noise variances.
VL, THI, Z, TME	Initial velocity, heading (in degrees), aircraft altitude, and time increment.
ETIME, TIME, FTIME	Elapsed time, time increment, flight time.
ETA	Initial launch angle.

2. <u>Subroutine INIT</u>

Symbol	Meaning
XA,YA,ZA,XDA,YDA,ZDA	Aircraft location and velocity in cartesian coordinate frame.
CK,CT,TH,V	Conversion factor (degrees to radians), converted heading angle, heading in degrees, velocity in meters/second.

3. <u>Subroutine NAVPCK</u>

Symbol	Meaning
CK,CT	Conversion factor (degrees to radians), converted heading.
XA,YA,ZA,XDA,YDA,ZDA,TH,V	Aircraft state vector.

4. <u>Subroutine MINIT</u>

Symbol	Meaning
YAW,PITCH	Missile command vector.
XMM,YMM,ZMM,XDMM,YDMM, ZDMM	Measured missile vector in position and velocity.
XM,YM,ZM,XDM,YDM,ZDM, XDDM,YDDM,ZDDM, HEAD,GAMMA,THET, THETD,THETDD	True missile vector.

5. <u>Subroutine PFIL</u>

Symbol	Meaning
X,Y	True target location.
QK,QM,K,M,J	Constants used for interpolation.
XS,YS,A	X variance, Y variance, noise value returned by GAUSS.
XT,YT,CA	Noisy target location, CPA in problem.

6. Subroutine MISAD

Symbol	Meaning
RHO,T,G	Air density, thrust, gravity.
REFAR	Aerodynamic reference area.
K1,K3,K4,K5	Drag, lift and moment constants.
K,YINC,ANG,VEL,VELSQ	Advance count, yaw increment, current missile heading, current velocity, current velocity squared.
DELTA, GAMMA, ALPHA	Control angle, missile axis angle, angle of attack.
D,QL,QM	Drag, lift, and moment forces.
QMASS,QIYY,QMULT	Missile mass, moment of inertia, constant relating mass and moment.

7. Subroutine REVCON

Symbol	Meaning
X,Y,Z	Work variables.
RPRI,RDPRI,T1,T2,VLXY	Work variables for developing the radar vector.
R,RD,T,P	True radar vector.

8. Subroutine RADAR2

Symbol	Meaning
C1	Bin size for 13-bit shaft encoder.
ITEMP,AN	Temporary variable used in the conversions, and noise value returned by GAUSS.
R,RD,T,P	Noisy, quantized radar vector.

9. Subroutine AFIL

Symbol	Meaning
NP	Switch to control filter initialization.
ZR1,ZR2,ZA1,ZD1	Measurements of range, range rate, azimuth, and depression angles.
XR1,XR2,XA1,XA2,XD1,XD2	Estimated values of range, range rate, azimuth, azimuth rate, depression, and depression rate.
GRG,GAZ,GDP	Range,azimuth, and depression filter gains.

10. Subroutine CORDCN

Symbol	Meaning
CSP,SNP,CST,SNT	Sine and cosine of ϕ and θ.
CSPSNT,CSPCST,SNPSNT, SNPCST	Cross products of sine and cosine values.
R,RD,T,TD,P,PD	Range, range rate, theta, theta rate, phi, and phi rate.
XMM,YMM,ZMM	Measured missile cartesian position.
XDMM,YDMM,ZDMM	Measured missile cartesian velocity.

11. Subroutine SHOOT

Symbol	Meaning
GAMMA, HEAD	Current missile velocity vector elevation and current missile heading.
ETA,A1	Desired missile velocity vector elevation and current LOS angle from the missile to the target.
YAW,PITCH	Missile command vector.

12. Subroutine PREDCT

Symbol	Meaning
QMASS,RHO,REFAR	Missile mass, air density, missile aerodynamic reference area.
K2,K3,V	Drag coefficients and missile velocity.
XP,YP,ZP	X and Y impact position predictions, and time until impact.

13. Subroutine NR

Symbol	Meaning
QMASS,RHO,REFAR,G,V	Missile mass, air density, missile aerodynamic reference area, gravity, and missile velocity.
K1,ALPHA	Drag coefficient and exponential argument.
A,B,C,TOLD,TNEW	Coefficients of prediction equations, last time estimate, new time estimate.
FT,FDT	Function value, and derivative.

14. Subroutine COMGEN

Symbol	Meaning
YAW,PITCH	Missile Command vector.
Numeric arguments	Conversion factors.

15. Subroutine DISGEN

Symbol	Meaning
XDF,YDF	Translated predicted impact point.
XTA,YTA	Used to determine rotation angle.
ALPH	Rotation angle.

XD,YD,ZD	Display points and time until impact.

16. Subroutine MISS

Symbol	Meaning
ZTM1,TI1,TI2	Last preimpact missile Z position, times which bracket impact time.
XIM1,YIM1,XI,YI	Last preimpact X and Y positions, and missile impact positions in X and Y.
ETI,FTI	Impact time as elapsed time and flight time.
K1,K2,QK2	Variables used to edit inaccuracies out of flight time clock.
XMS,YMS,XT,YT	X and Y miss distances, and last estimated target position.

C. INTERROUTINE COMMUNICATION

Interroutine communication is carried out throughout the program by means of labeled FORTRAN COMMON blocks. These blocks are designed in such a manner that variables relating closely to each other have been grouped into sections named to reflect their functions. The block names are given below along with the variables included and a description of the function of the block.

1. Blank COMMON

The variables included in blank COMMON are IX, ETIME, TIME, and FTIME. IX was originally the seed value for the GAUSS routine which produces random numbers. It was not retained when a change in random number generators was made.

ETIME, TIME, and FTIME are the problem elapsed time, time increment, and missile flight time respectively.

2. Block TGPRD

The variables included in the block TGPRD are the cartesian location of the target and the X and Y predicted impact points of the missile. ZP is the time until impact.

3. Block SIGMAS

This block contains the values of the sigmas to be used in the radar to introduce noise in the measurements. These values are set in the main line program.

4. Block GAINS

The computed filter gains used by the active radar filter are stored in this dimensioned block. The block provides storage for 120 range filter gains and 60 azimuth and depression filter gains each.

5. Block MISLM

This block contains the values of measured missile position and velocity developed by the missile tracking routine.

6. Block ACRFT

The values of aircraft position and velocity at each time k are held in this block. Also this block contains the aircraft velocity in meters per second and aircraft heading in degrees.

7. Block MISLT

This vector contains all values which pertain to the true missile. The vector contains position, velocity, and acceleration in cartesian coordinates; missile axis angle,

angular velocity, and angular acceleration; and velocity vector heading and elevation angle. This is updated each sample period to reflect the true state of the missile.

8. Block COMND

The commands to the missile in yaw and pitch comprise the two elements of this block. The yaw values are up to twenty degrees per second in either direction and pitch values range from plus to minus forty-five degrees in control deflection.

9. Block SIGS

The values of variance which are used in the passive location routine PFIL are transferred to the routine by this block. Each of the values is dimensioned to accept fifty-nine values of variance.

10. Block RADAR

This block is used to transfer the values of the radar information through all of the routines of the missile tracking portion of the program. The six elements of the vector are range, range rate, azimuth, azimuth rate, depression, and depression rate.

D. PROGRAMMING "TRICKS"

Only one programming irregularity exists in the simulation program. In the subroutine PFIL the variable ZP in the COMMON block TGPRD is used as a switch to allow the routine to hold the true target location after the first call. The variable ZP was chosen since it was already in the COMMON block and since it is not used by any other routine in the passive phase of the program.

```
      COMMON IX,ETIME,TIME,FTIME
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      COMMON /SIGMAS/ ARNG,ARNGD,AAZ,ADP
      COMMON /GAINS/ GRG(30 ,4),GAZ(30 ,2),GDP(30 ,2) ,KBQ
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
      COMMON /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THET
     1DD,HEAD,GAMMA,K
      COMMON /COMND/ YAW,PITCH
      COMMON /SIGS/ XSIG(59),YSIG(59)
C     MAIN LINE PROGRAM THESIS
      READ(5,101)GBQ
      KBQ=GBQ
      DO 10 I=1,KBQ
   10 READ(5,100) GRG(I,1),GRG(I,2),GRG(I,3),GRG(I,4)
      DO 11 I=1,KBQ
   11 READ(5,100) GAZ(I,1),GAZ(I,2),GDP(I,1),GDP(I,2)
      READ(5,100) (XSIG(I),I=1,59)
      READ(5,100) (YSIG(I),I=1,59)
      READ(5,101) RUNS
      RNS=RUNS+1.
   15 READ(5,101) VL,THI,Z,TME
      ZP=0.
      ETA=22.*3.1415926535/180.
      NP=1
      KK=1
      SMU=0.
      ARNG=10.
      ARNGD=3.16
      AAZ=.02
      ADP=.02
      KR=RNS-RUNS
      CALL INIT(VL,THI,Z,TME)
      READ(5,101) XT,YT,ZT
      WRITE(6,200) KR
      CALL POTSET
```

```
      PAUSE 77
   61 IF(TEST(7)) 60,61,61
   60 CALL SETLINES(7,-1)
      CALL SETLINES(7,1)
      IF (ETIME.GT.59.) GO TO 42
      CALL NAVPCK
      CALL PFIL
    1 WRITE(30,91) XT,YT,ETIME
   41 IF(ETIME.LT.59.)  GO TO 61
    5 QJ=I1
      I1=1
      WRITE(30,92) QJ,VL,THI,Z,TME
   19 WRITE(6,201) KR
      WRITE(6,202)
      WRITE(6,203)
      WRITE(6,204)
   20 CALL SHOOT(ETA)
   40 CALL NAVPCK
      CALL MISAD
   25 CALL REVCON
      CALL RADAR2
      CALL AFIL(NP)
      CALL CORDCN
      IF(ZMM-ZT.LT.0.) GO TO 30
      CALL PREDCT
      CALL COMGEN
      CALL DISGEN
    3 WRITE(31,93)XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THETDD,
     1HEAD,GAMMA,XMM,YMM,ZMM,XDMM,YDMM,ZDMM,XP,YP,YAW,PITCH,ETIME,FTIME
      GO TO 61
   42 IF(FTIME.LT.30.) GO TO 20
      CALL RESET
      GO TO 40
   30 CONTINUE
   31 CALL MISS
      END FILE 2
```

```
      RUNS=RUNS-1.
      IF(RUNS.GT.0.) GØ TØ 15
      END FILE 2
   35 WRITE(6,103)
   91 FØRMAT(1H ,3(E12.6,2X))
   92 FØRMAT(1H1,'CYCLES = ',F8.1,' A/C VELØCITY = ',F8.2,' A/C HEADING
     1= ',F8.2,' A/C ALTITUDE = ',F10.2,' TIME INCREMENT = ',F8.2)
   93 FØRMAT(1H0,9(E12.6,2X),/,1X,5(E12.6,2X),/,1X,8(E12.6,2X),/,1X,4(E1
     12.6,2X))
  100 FØRMAT(4E20.8)
  101 FØRMAT(4F15.2)
  103 FØRMAT(1H1,'END ØF RUNS.')
  200 FØRMAT(1H1,'PASSIVE RUN ',I1,/,1H ,'XT',14X,'YT',12X,'ETIME',//)
  201 FØRMAT(1H1,'ACTIVE RUN ',I1,/,1H ,'XM',12X,'YM',12X'ZM',12X,'XDM',
     111X,'YDM',11X,'ZDM'11X,'XDDM',10X,'YDDM',10X,'ZDDM')
  202 FØRMAT(1H ,'THETA',9X,'THETA DØT',5X,'THETA DØT DØT',1X,'HEADING',
     17X,'VL PITCH')
  203 FØRMAT(1H ,'XMM',11X,'YMM',11X,'ZMM',11X,'XDMM',10X,'YDMM',10X,'ZD
     1MM',10X,'XP',12X,'YP')
  204 FØRMAT(1H ,'YAW',11X,'PITCH',9X,'ELAPSED TIME',2X,'FLIGHT TIME',/)
      STØP
      END
```

```
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.
      DO 50 I=1,12
      CALL RANDOM(W)
   50 A=A+W
      V=(A-6.0)*S+AM
      RETURN
      END
```

```
      SUBROUTINE RANDOM(W)
      DIMENSION R(30)
      DATA I/10/
      DATA R/
     C   .150781319000088,   .914453206998587,   .514453231837251,
     C   .898828280478483,   .815625252795143,   .612891073171340,
     C   .838672715504799,   .397655462456896,   .273045296104100,
     C   .800387462008075,   .779681167623493,   .810924827368580,
     C   .094115270407201,   .029246154575957,   .416304796039185,
     C   .855265827125549,   .546078512492385,   .888641380963235,
     C   .682751490840018,   .528002958235447,   .623584015538654,
     C   .367871127691614,   .557617224250861,   .786328164656879,
     C   .241015667790634,   .295703170922934,   .698437549086520/
C     THESE VALUES WERE OBTAINED BY RUNNING THE OLD SELF-INITIALIZING
C     VERSION OF RANDOM 100 TIMES AND TAKING THE RESULTING VALUES IN R
C     THIS VERSION NEVER USES ITS ARGUMENT AS AN INPUT, AN ADDITIONAL
C     SAFEGUARD. RANDOM PRODUCES VALUES BETWEEN 0 AND 1.
    2 J=I
      I=I+1
      IF(I  .GT.  30) I=1
      W=R(J)-R(I)
      IF(W .LE. 0.0) W=W+.999999999837
      R(I)=W
      RETURN
      END
```

```
SUBROUTINE NAVPCK
COMMON IX,ETIME,TIME,FTIME
COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
ETIME=ETIME+TIME
XA=XA+XDA*TIME
YA=YA+YDA*TIME
RETURN
END
```

```
      SUBRØUTINE MISS
      CØMMØN IX,ETIME,TIME,FTIME
      CØMMØN /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THET
     1DD,HEAD,GAMMA,K
      CØMMØN /TGPRD/ XT,YT,ZT,XP,YP,ZP
C  CØMPUTING MISS DISTANCES BY FINDING EXACT IMPACT TIME.
   31 ZTM1=ZM-ZDDM*TIME*TIME/2.-ZDM*TIME
      TI1=(-ZDM+SQRT(ZDM*ZDM+19.6*(ZTM1-ZT)))/(-9.8)
      TI2=(-ZDM-SQRT(ZDM*ZDM+19.6*(ZTM1-ZT)))/(-9.8)
      IF(TI2.GT.TI1) GØ TØ 10
      TI=TI1
      GØ TØ 20
   10 TI=TI2
C  BACKING ØFF X AND Y PØSITIØNS.
   20 XIM1=XM-XDDM*TIME*TIME/2.-XDM*TIME
      YIM1=YM-YDDM*TIME*TIME/2.-YDM*TIME
C  CØMPUTING X AND Y IMPACT PØINTS.
      XI=XDDM*TI*TI/2.+XDM*TI+XIM1
      YI=YDDM*TI*TI/2.+YDM*TI+YIM1
C  CØMPUTING IMPACT TIMES.
      ETI=ETIME-1.+TI
      FTI=FTIME-1.+TI
C  EDITING INACCURACIES ØUT ØF FLIGHT TIME CLØCK.
      K1=FTI
      K2=ETI
      QK2=ETI-K2
      FTI=K1+QK2
C  CØMPUTING MISS DISTANCES.
      XMS=XT-XI
      YMS=YT-YI
      WRITE(6,100)ETI,FTI
  100 FØRMAT(1H1,'MISSILE IMPACTED ',F7.1,' SECØNDS AFTER PRØBLEM START
     1AND ',F7.1,' SECØNDS AFTER LAUNCH.')
      WRITE(6,105)XI,XT,XMS
  105 FØRMAT(1H0,'X IMPACT AT ',F10.2,' METERS.',/,' TARGET X AT ',F10.2
     1,' METERS.',/,' X MISS WAS ',F10.2,' METERS.')
```

```
      WRITE(6,105)XI,XT,XMS
  105 FORMAT(1HO,'X IMPACT AT ',F10.2,' METERS.',/,' TARGET X AT ',F10.2
     1,' METERS.',/,' X MISS WAS ',F11.2,' METERS.')
      WRITE(6,115)
      WRITE(6,110)YI,YT,YMS
  110 FORMAT(1HO,'Y IMPACT AT ',F10.2,' METERS.',/,' TARGET Y AT ',F10.2
     1,' METERS.',/,' Y MISS WAS ',F11.2,' METERS.')
      WRITE(6,115)
  115 FORMAT(1HO,120('*'))
      WRITE(6,115)
      TMIS=SQRT(XMS*XMS+YMS*YMS)
      WRITE(6,120) TMIS
  120 FORMAT(1HO,'TOTAL MISS DISTANCE WAS ',F10.2,' METERS.')
      WRITE(6,115)
      RETURN
      END
```

```
      SUBROUTINE INIT(VL,THI,Z,TME)
      COMMON IX,ETIME,TIME,FTIME
      COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
C  SETTING INITIAL POSITION IN METERS.
      XA=0.
      YA=0.
      ZA=Z
C  SETTING INITIAL VELOCITIES IN M/SEC.
      CK=3.1415926535/180.
      CT=THI*CK
      TH=THI
      XDA=VL*SIN(CT)
      YDA=VL*COS(CT)
      ZDA=0.
      V=VL
C  ZEROING TIMES AND SETTING TIME INCREMENT.
      FTIME=0.
      ETIME=0.
      TIME=TME
      RETURN
      END
```

```
      SUBRØUTINE PFIL
      CØMMØN IX,ETIME,TIME,FTIME
      CØMMØN /SIGS/ XSIG(59),YSIG(59)
      CØMMØN /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
      CØMMØN /TGPRD/ XT,YT,ZT,XP,YP,ZP
      QKJ=36.*1760./39.37
      IF(ZP.NE.0.)  GØ TØ 10
      X=XT
      Y=YT
      ZP=99.
   10 IF(ETIME.LT.4.) RETURN
      CA=SQRT(XT*XT+YT*YT)*SIN(3.1415926535/2.-TH*3.1415926535/180.-ATAN
     12(YT,XT))
      QM=2./TIME
      M=QM
      K=ETIME/TIME
      K=MØD(K,M)
      QK=K
      J=((ETIME+2.)/2.)-2.
      K=J+1
      XS=(XSIG(J)+((XSIG(K)-XSIG(J))*QK/QM))*CA/30.
      XS=SQRT(XS)
      CALL GAUSS(IX,XS,0.,A)
      XT=X+A
      YS=(YSIG(J)+((YSIG(K)-YSIG(J))*QK/QM))*CA/30.
      YS=SQRT(YS)
      CALL GAUSS(IX,YS,0.,A)
      YT=Y+A
      RETURN
      END
```

```
      SUBROUTINE MISAD
      COMMON IX,ETIME,TIME,FTIME
      COMMON /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,
     1THETDD,HEAD,GAMMA,K
      COMMON /COMND/ YAW,PITCH
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      REAL K1,K2,K3,K4,K5
C
C     - YAW = LEFT YAW.  + YAW = RIGHT YAW.  ZERO YAW = NO YAW.
C     -PITCH = DOWN PITCH. +PITCH= UP PITCH. ZERO PITCH = NO PITCH
C     COMPUTING AND SETTING CONSTANTS.
C
      RHO=.08018*.4536/.02832
      QMASS=453.6
      T=3000.*4.448
      G=9.8
      K1=.025/4.5
      K3=.01
      K4=.004
      K5=.02*2.5
      QIYY=150.*453.6/200.
      QMULT=QIYY/QMASS
      QJJ=1.
      QLL=2.
      K=TIME/.1
      DO 10 I=1,K
      VEL=SQRT(XDM*XDM+YDM*YDM+ZDM*ZDM)
      IF(YAW*VEL.GT.98.)  YAW=98./VEL
      IF(YAW*VEL.LT.-98.)  YAW=-98./VEL
      YINC=YAW*.1
      RD=SQRT(XDM*XDM+YDM*YDM)
      ANG=ATAN2(XDM,YDM)
      HEAD=ANG+YINC
      XDM=RD*SIN(HEAD)
      YDM=RD*COS(HEAD)
      VELSQ=VEL*VEL
```

```
      DELTA=PITCH
      GAMMA=ATAN2(ZDM,RD)
      ALPHA=THET-GAMMA
      Q=RHØ*VELSQ
      CF=K3*DELTA*Q
      D=K1*Q
      QL=K5*Q*ALPHA
      QM=-QJJ*(QL+D*SIN(ALPHA))+CF*QLL-K4*Q*THETD
      IF(FTIME.GT.30.)  GØ TØ 20
      QMASS=(1000.-12.*FTIME)*.4536
      GØ TØ 25
   20 T=0.
      QMASS=640.*.4536
   25 QIYY=QMASS*QMULT
      CØSG=CØS(GAMMA)
      SING=SIN(GAMMA)
      CØST=CØS(THET)
      SINT=SIN(THET)
      RDDM=(T*CØST-QL*SINT-D*CØSG)/QMASS
      XDDM=RDDM*SIN(HEAD)
      YDDM=RDDM*CØS(HEAD)
      ZDDM=(T*SINT+QL*CØST-D*SING-QMASS*G)/QMASS
      XDØ=XDM
      YDØ=YDM
      ZDØ=ZDM
      XDM=XDDM*.1+XDØ
      YDM=YDDM*.1+YDØ
      ZDM=ZDDM*.1+ZDØ
      XM=XDDM*.005+XDØ*.1+XM
      YM=YDDM*.005+YDØ*.1+YM
      ZM=ZDDM*.005+ZDØ*.1+ZM
      THETDD=QM/QIYY
      TDØ=THETD
      THETD=THETDD*.1+TDØ
      THET=THETDD*.005+TDØ*.1+THET
      FTIME=FTIME+.1
```

```
10 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE SHOOT(ETA)
      COMMON IX,ETIME,TIME,FTIME
      COMMON /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THET
     1DD,HEAD,GAMMA,K
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /COMND/ YAW,PITCH
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      IF(FTIME.GT.0.)  GO TO 10
      CALL MINIT
   10 IF(ETA-GAMMA.LT..01.AND.ETA-GAMMA.GT.-.01)  PITCH=0.
      IF(ETA-GAMMA.GT..01)  PITCH= .5
      IF(ETA-GAMMA.LT.-.01)  PITCH=-.5
      A1=ATAN2(XT-XMM,YT-YMM)
      IF(HEAD-A1.LT..01.AND.HEAD-A1.GT.-.01)  YAW=0.
      IF(HEAD-A1.LT.-.01)  YAW=.1
      IF(HEAD-A1.GT..01)  YAW=-.1
      RETURN
      END
```

```
      SUBROUTINE MINIT
      COMMON /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THET
     1DD,HEAD,GAMMA,K
      COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /COMND/ YAW,PITCH
C   LAUNCHING THE MISSILE.
      XMM=XA
      YMM=YA
      ZMM=ZA
      XM=XA
      YM=YA
      ZM=ZA
      XDMM=XDA+1.
      YDMM=YDA+1.
      ZDMM=ZDA+1.
      XDM=XDA
      YDM=YDA
      ZDM=ZDA
      YAW=0.
      PITCH=0.
      HEAD=TH*3.1415926535/180.
      GAMMA=0.
      THET=0.
      THETD=0.
      THETDD=0.
      RETURN
      END
```

```
      SUBROUTINE REVCON
      COMMON /MISLT/ XM,YM,ZM,XDM,YDM,ZDM,XDDM,YDDM,ZDDM,THET,THETD,THET
     1DD,HEAD,GAMMA,K
      COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
      COMMON /RADAR/ R,RD,T,TD,P,PD
C   COMPUTING THE RANGE AND THE AZIMUTH AND DEPRESSION ANGLES.
   25 X=XM-XA
      Y=YM-YA
      Z=ZM-ZA
      RPRI=SQRT(X*X+Y*Y)
      T=ATAN2(X,Y)
      P=ATAN2(Z,RPRI)
      R=SQRT(X*X+Y*Y+Z*Z)
C   COMPUTING THE R DOT COMPONENT.
      X=XDM-XDA
      Y=YDM-YDA
      Z=ZDM-ZDA
      RDPRI=SQRT(X*X+Y*Y)
      T1=ATAN2(YM-YA,XM-XA)
      T2=ATAN2(Y,X)
      VLXY=RDPRI*COS(T2-T1)
      RD=VLXY*COS(P)
      RETURN
      END
```

```
      SUBROUTINE RADAR2
      COMMON IX,ETIME,TIME,FTIME
      COMMON /RADAR/ R,RD,T,TD,P,PD
      COMMON /SIGMAS/          ARNG,ARNGD,AAZ,ADP,SMU
C  BIN SIZE FOR 13 BIT SHAFT ENCODER.
      C1=2.*3.1415926535/8192.
C  MAKING ANGLES NOISY.
      CALL GAUSS(IX,AAZ,SMU,AN)
      T=T+AN
      CALL GAUSS(IX,ADP,SMU,AN)
      P=P+AN
C  QUANTIZING THETA AND PHI.
      ITEMP=T/C1
      T=ITEMP*C1+C1/2.
      ITEMP=P/C1
      P=ITEMP*C1+C1/2.
C  MAKING RANGE AND RANGE RATE NOISY.
      CALL GAUSS(IX,ARNG,SMU,AN)
      R=R+AN
      CALL GAUSS(IX,ARNGD,SMU,AN)
      RD=RD+AN
C  GUANTIZING RANGE INTO 10 METER BINS.
C  GUANTIZING RANGE RATE INTO 2.5 M/SEC. BINS.
      ITEMP=R/10.
      R=ITEMP*10.+5.
      ITEMP=RD/2.5
      RD=ITEMP*2.5+1.25
      RETURN
      END
```

```
      SUBROUTINE AFIL(NP)
      COMMON IX,ETIME,TIME,FTIME
      COMMON /RADAR/ R,RD,T,TD,P,PD
      COMMON /GAINS/ GRG(30 ,4),GAZ(30 ,2),GDP(30 ,2),KBQ
      GO TO (10,20),NP
   10 XR1=0.
      XR2=0.
      XA1=0.
      XA2=0.
      XD1=0.
      XD2=0.
      NP=2
   20 ZR1=R
      ZR2=RD
      ZA1=T
      ZD1=P
C  SMOOTHING (FILTERING) THE NOISY, QUANTITIZED RADAR INFORMATION.
      KQ=FTIME/TIME+.1
      IF(KQ.GT.KBQ)  KQ=KBQ
      R=XR1+(GRG(KQ,1)*(ZR1-XR1)+GRG(KQ,2)*(ZR2-XR2))
      RD=XR2+(GRG(KQ,3)*(ZR1-XR1)+GRG(KQ,4)*(ZR2-XR2))
      XR1=R+RD*TIME
      XR2=RD
      T=XA1+GAZ(KQ,1)*(ZA1-XA1)
      TD=XA2+GAZ(KQ,2)*(ZA1-XA1)
      XA1=T+TD*TIME
      XA2=TD
      P=XD1+GDP(KQ,1)*(ZD1-XD1)
      PD=XD2+GDP(KQ,2)*(ZD1-XD1)
      XD1=P+PD*TIME
      XD2=PD
      RETURN
      END
```

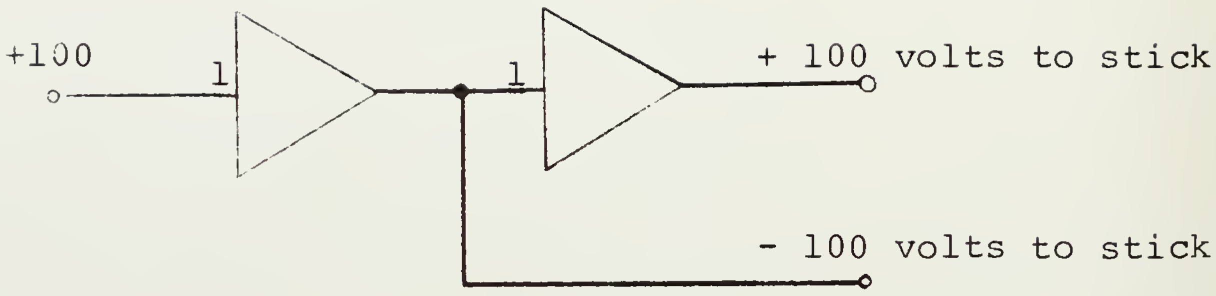
```
      SUBROUTINE CORDCN
      COMMON /RADAR/ R,RD,T,TD,P,PD
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /ACRFT/ XA,YA,ZA,XDA,YDA,ZDA,TH,V
C  COMPUTE AND STORE SINGLE TRIG FUNCTIONS.
      CSP=COS(P)
      SNP=SIN(P)
      CST=COS(T)
      SNT=SIN(T)
C  COMPUTE AND STORE CROSS FUNCTIONS.
      CSPSNT=CSP*SNT
      CSPCST=CSP*CST
      SNPSNT=SNP*SNT
      SNPCST=SNP*CST
C  COMPUTE AND STORE POSITIONALS.
      XMM=R*CSPSNT+XA
      YMM=R*CSPCST+YA
      ZMM=R*SNP+ZA
C  COMPUTING THE VELOCITY TERMS.
      XDMM=RD*CSPSNT-R*SNPSNT*PD+R*CSPCST*TD+XDA
      YDMM=RD*CSPCST-R*SNPCST*PD-R*CSPSNT*TD+YDA
      ZDMM=R*CSP*PD+RD*SNP+ZDA
      RETURN
      END
```

```
      SUBROUTINE PREDCT
      COMMON IX,ETIME,TIME,FTIME
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      REAL K2,K3
   15 QMASS=640.*.4536
      RHO=.08018/.02832*.4536
   10 TI=100.
   20 CALL NR(TI)
      K2=RHO*ABS(XDMM)
      K3=RHO*ABS(YDMM)
      XP=XMM+(QMASS*XDMM/K2)*(1.-EXP(-K2*TI/QMASS))
      YP=YMM+(QMASS*YDMM/K3)*(1.-EXP(-K3*TI/QMASS))
      ZP=TI
      RETURN
      END
```

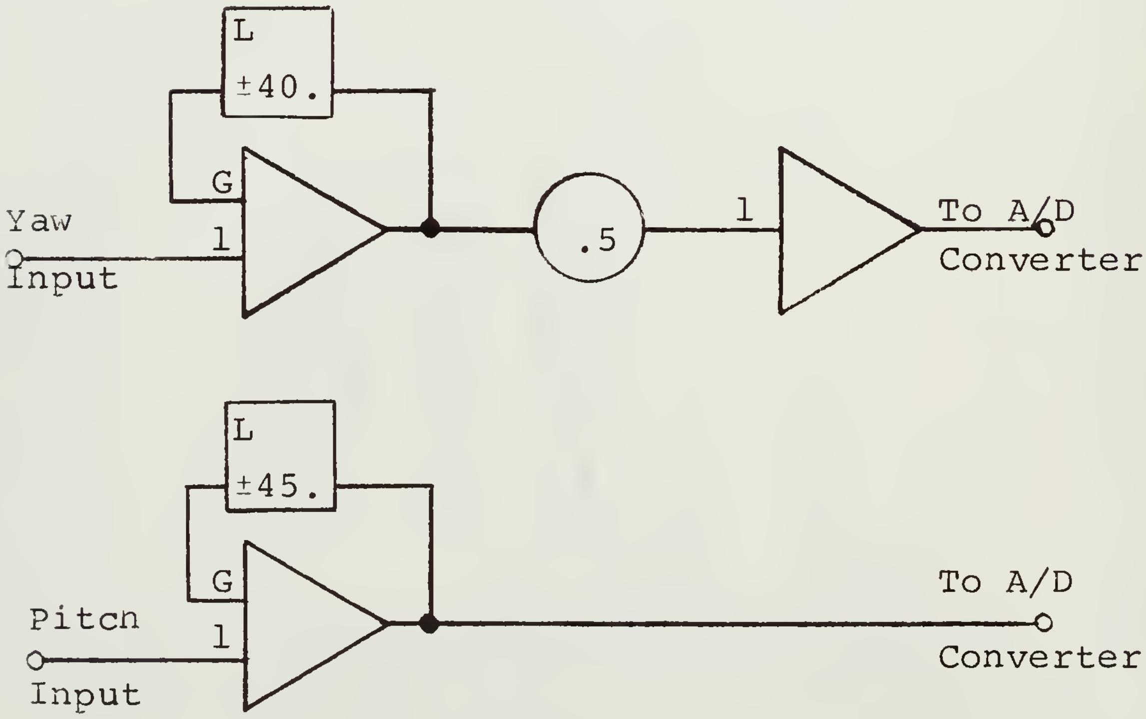
```
      SUBROUTINE NR(TNEW)
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      REAL K1
      RHO=.08018/.02832*.4536
      G=9.8
      QMASS=640.*.4536
      K1=RHO*ABS(ZDMM)
      ALPHA=K1/QMASS
      A=QMASS*ZDMM/K1+QMASS*QMASS*G/(K1*K1)
      B=QMASS*G/K1
      C=ZMM-ZT
      TOLD=TNEW
   10 FT=A*(1.-EXP(-ALPHA*TOLD))-B*TOLD+C
      FDT=ALPHA*A*EXP(-ALPHA*TOLD)-B
      TNEW=TOLD-FT/FDT
      IF(ABS(TNEW-TOLD).LE..1)  GO TO 20
      TOLD=TNEW
      GO TO 10
   20 RETURN
      END
```

```
SUBROUTINE COMGEN
COMMON IX,ETIME,TIME,FTIME
COMMON /COMND/ YAW,PITCH
IF(FTIME.LT.30.) RETURN
CALL ADK( 6,YAW, 7,PITCH)
YAW=YAW*5.*3.1415926535/9.
PITCH=PITCH*5.*3.1415926535/9.
RETURN
END
```

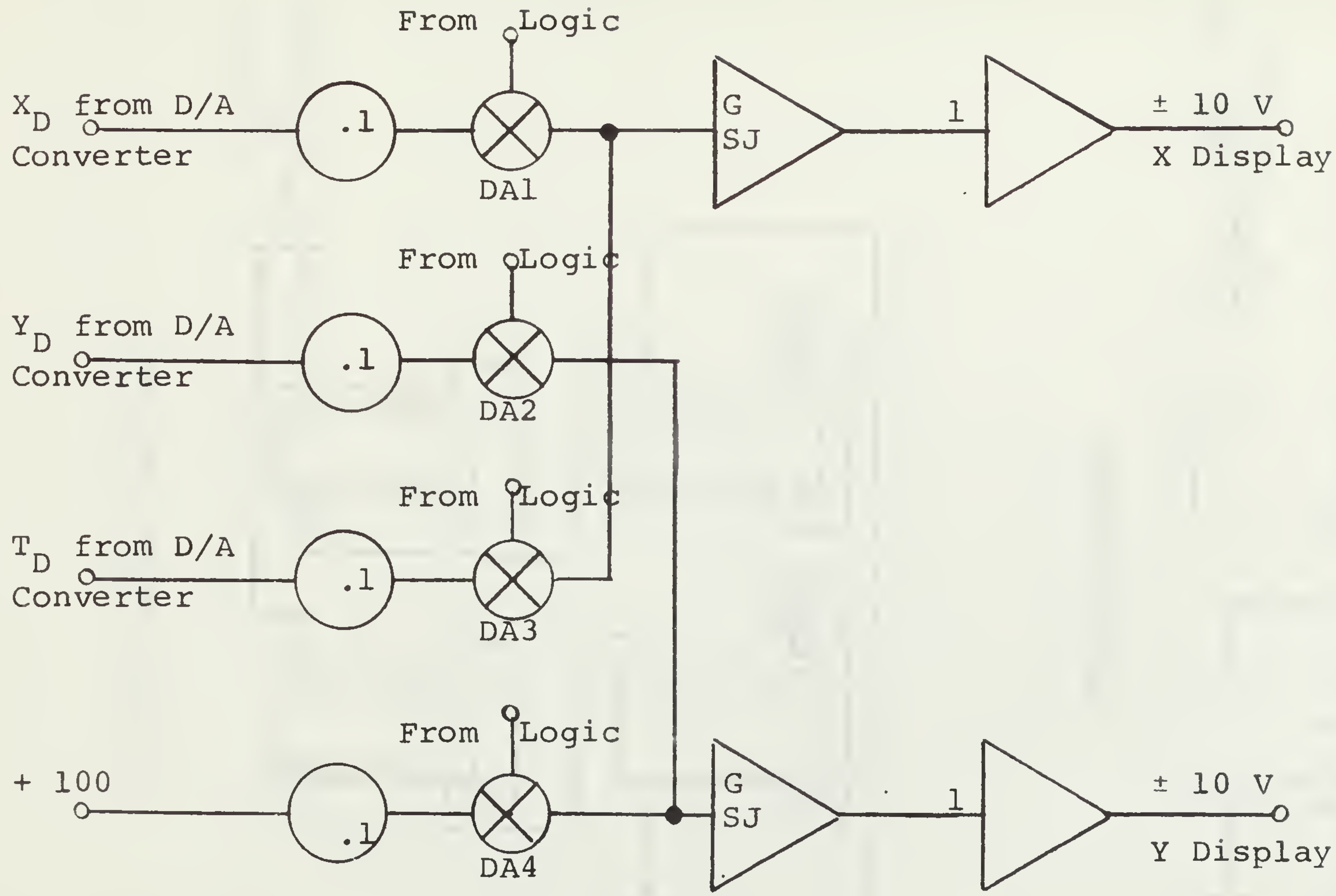
```
      SUBROUTINE DISGEN
      COMMON /TGPRD/ XT,YT,ZT,XP,YP,ZP
      COMMON /MISLM/ XMM,YMM,ZMM,XDMM,YDMM,ZDMM,KK
      XDF=XP-XT
      YDF=YP-YT
      XTA=XT-XMM
      YTA=YT-YMM
      ALPH=ATAN2(XTA,YTA)
      XTA=XTA/20000.
      YTA=YTA/20000.
      CALL DAC(4,XTA,5,YTA)
      CA=COS(ALPH)
      SA=SIN(ALPH)
    5 XD=XDF*CA-YDF*SA
      YD=XDF*SA+YDF*CA
      XD=XD/10000.0
      IF(XD.GT.1.)  XD= .999999999
      IF(XD.LT.-1.)  XD=-.999999999
      YD=YD/10000.0
      IF(YD.GT.1.)  YD= .999999999
      IF(YD.LT.-1.)  YD=-.999999999
      ZD=ZP/100.0
      IF(ZD.GT.1.)  ZD= .999999999
      IF(ZD.LT.-1.)  ZD=-.999999999
      CALL DAC(1,XD,2,YD,3,ZD)
      RETURN
      END
```



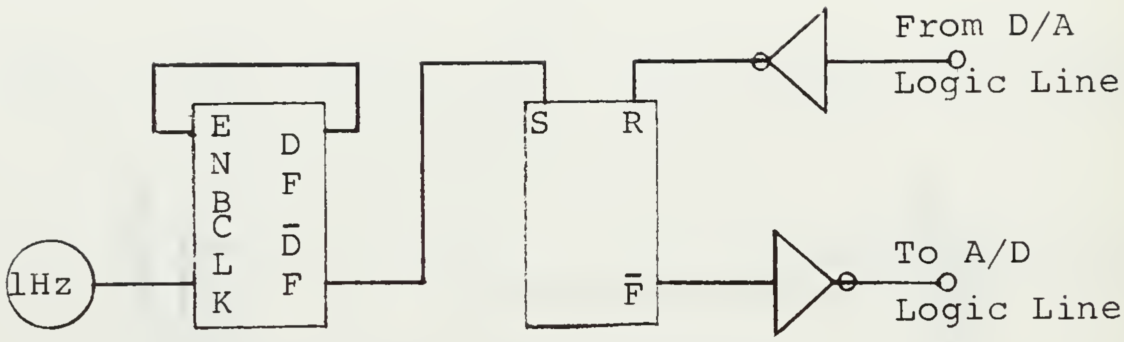
CONTROL STICK POWER SUPPLY



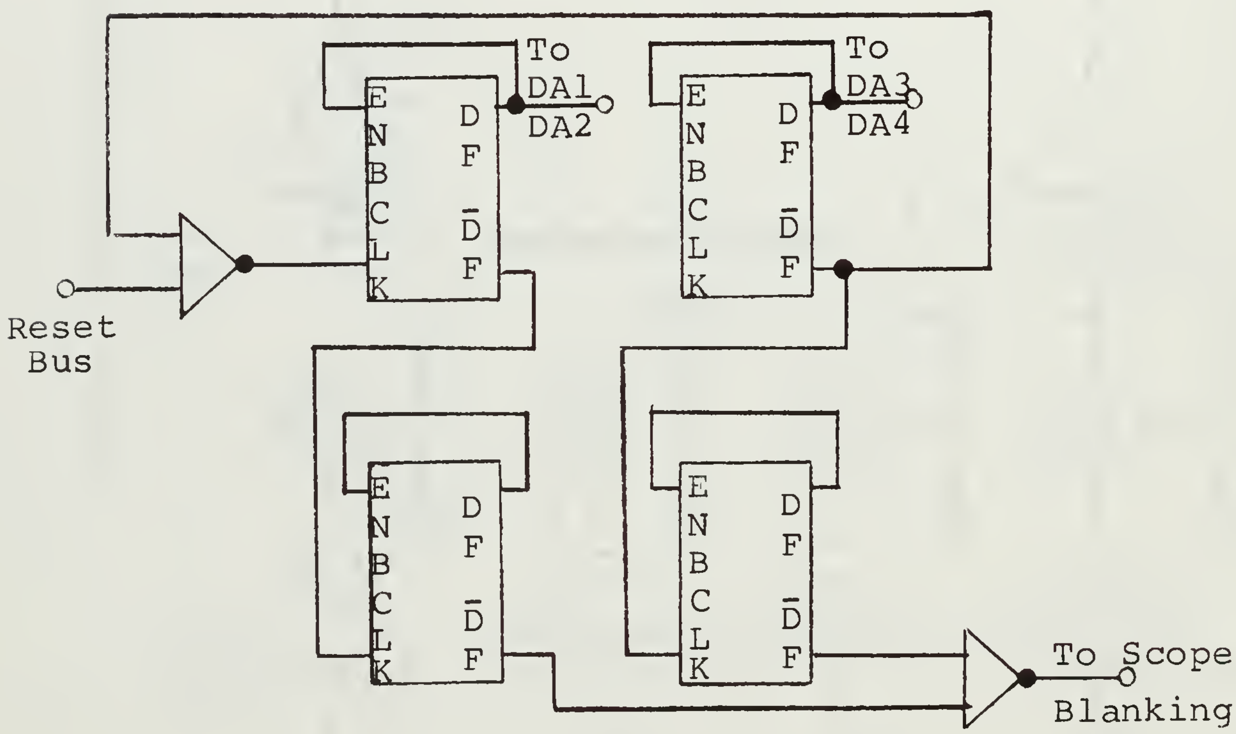
YAW/PITCH PROCESSING BEFORE A/D CONVERSION



DISPLAY SCALING AND SWITCHING



CLOCK SECTION



DISPLAY DRIVER SECTION

BIBLIOGRAPHY

1. Naval Postgraduate School, NPS-52DE8041A, Estimation Algorithms For Location Of Stationary Radiation Sources By Bearing Measurements From Moving Aircraft, by James S. Demetry, p. 10-12, 15 April 1969.

2. Lee, Robert C. K., Optimal Estimation, Identification and Control, p. 38-41, 43-46, MIT Press, 1964.

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13. ABSTRACT

This paper investigates the feasibility of an air-to-surface missile delivery system employing impact prediction to assist a human operator in controlling the system. The impact prediction information was used to drive a display whicn the operator used in controlling the flight of the weapon. It was desired to find whether impact prediction proved useful in weapons control.

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